On the Dynamics, Equilibria, and Economics of Gender Balance at Naturist Venues

James Junghanns

ORCID 0009-0004-3175-8962

Revised Preprint DOI 10.5281/zenodo.20044325

May 5, 2026

Abstract

We propose a dynamical systems approach to model the demographic evolution of naturist venues. A coupled ODE system yields a characteristic polynomial $P(r)$ whose root structure determines whether a venue is stable, exhibits a tipping point, or is irreversibly doomed. Under male indifference to gender imbalance (Case A), a critical ratio r_1 separates demographic collapse from recovery; restoring male sensitivity (Case B) relaxes viability conditions and provides an endogenous recovery mechanism. Exact phase-space trajectories are obtained by partial-fraction decomposition, and a Sundman temporal desingularisation reduces the ratio dynamics to a polynomial flow in dual time. The "no single males" entry rule is modelled as a binding constraint; Karush-Kuhn-Tucker conditions show that the global revenue optimum is explicitly identifiable, with the shadow price of female attendance equal to her own access fee plus the male slot her presence unlocks, providing a rigorous economic justification for cross-subsidisation and showing that revenue and welfare maximisation converge on the same strategy. A Voronoi percolation analogy motivates a female-fraction connectivity benchmark of $\mathcal{P}_c = 1/2$. Microfoundations derive the ODE as the mean-field limit of individually rational discrete-choice decisions; a scalar best-response map identifies its fixed points with the roots of $P(r)$ and its self-correction condition with ODE stability. A theorem establishes that $\kappa^* = 1$ (one admitted male per admitted female) is the unique revenue-maximising proportional quota subject to the connectivity benchmark, with the KKT shadow prices and the constrained ODE substitution following as corollaries.

Keywords:

gender ratio dynamics, naturist venues, characteristic polynomial, KKT shadow prices, mean-field limit, Voronoi percolation

MSC2020:

34D20, 34C60, 37N40, 91B16, 91B32, 60J28, 60D05, 68Q60

JEL:

C61, D11, D62, J11, L83, Z13

1. Introduction

1.1. Objectives

This paper aims to be a foundational contribution: it treats naturist venues as legitimate objects of study by mathematical social sciences and, to the authors' best knowledge, provides the first explicit dynamical and economic analysis of gender-balance feedback, stability, and quota policy in these settings. The mathematical methods are sound and varied, and well-tested. What is novel is the combination and that they are being employed jointly to model a phenomenon that has hitherto been ignored or passed over.

1.2. Roadmap

Part I: ***The Aggregate Model*** (Sections 2–4) develops the aggregate dynamical-systems account of venue demographics. The characteristic polynomial $P(r)$ and its roots organise everything: viability, tipping, phase trajectories, finite-time collapse, and the shadow-price economics of the quota constraint, all as consequences of a postulated ODE system.

Part II: ***Microfoundations and Rational Agency*** (Sections 5–6) deepens the analysis. Section 5 motivates why female social-graph connectivity matters for comfort and introduces the percolation benchmark. Section 6 supplies individual-decision microfoundations: the system of ODEs is derived as the mean-field limit of rational discrete-choice decisions; a scalar best-response map connects its fixed points to the roots of $P(r)$; and the optimal proportional quota $\kappa^* = 1$ is derived from first principles rather than assumed.

1.3. Scope and limitations

We model venue demographics using two coarse categories labelled '*female*' and '*male*'. These terms coincide with those used broadly within the naturist community; they denote sex-based demographic headcounts as these are not taboo and are manifest. The categorical partition is coarse; heterogeneity within each category is absorbed into reduced-form coefficients and not modelled explicitly, and the paper's conclusions concern only the dynamics implied by this partition. In practice, few communities are as accommodating of self-determination, or as far from active discrimination, as the naturist one. When we describe '*couples*' as stabilising via $\varphi_2 \approx 0$, the operative mechanism is social anchoring and reduced exposure to unwanted attention, which extends naturally beyond heterosexual pairing. Parameters such as φ_2 are reduced-form perceptions of risk shaped by norms, reputation, and enforcement; policy can shift these as well as states.

The intention is not to 'borrow' naturism as a colourful illustration of generic participation dynamics. We intend to model naturist venues on their own terms, with the same matter-of-fact mathematical posture that is routine for clubs, platforms, and other composition-sensitive social environments, each with their own particularities and nuances.

1.3.1. Terminology, scope of categories, and inclusion

Throughout this paper the terms '*female*' and '*male*' refer exclusively to observed demographic headcounts at naturist venues, as self-reported or visually recorded by venue operators. These are coarse binary categories that coincide with those used in everyday naturist practice and in the data-collection protocols of Appendices A and B. The model does not claim ontological status for these categories, nor does it address gender identity, non-binary participation, or intersex conditions; such heterogeneity is absorbed into the reduced-form coefficients φ_i and μ_i. The same modelling posture is standard in mathematical sociology when studying ratio-dependent participation (*e.g.*, Schelling-type models of residential segregation [1]). Same-sex couples and same-sex attraction are likewise absorbed into the same reduced-form parameters; extending the system with explicit orientation-specific coefficients would add parameters without adding explanatory power at the current level of aggregation.

1.4. Social Science: observables and falsifiability

The Appendices are devoted to actionable methods for the collection of quantitative data and the interpretation of observables that could be compatible with the predictions made in this paper or falsify the model that underpins them. We believe that it is an important aspect when presenting a novel model of any phenomenon as it is what distinguishes a successful model from a compelling narrative.

Symbol	**Meaning**	**First used**
f, m	Female, male headcounts	(1)
$r = f/m$	Gender ratio	(3)
φ_1, φ_2	Female entry/exit sensitivity coefficients	(1)
μ_1, μ_2	Male entry/exit sensitivity coefficients	(1)
$P(r)$	Characteristic polynomial	(9)
r_1	Unstable (tipping) equilibrium ratio	(26)
r_2	Stable (natural) equilibrium ratio	(26)
$\rho_1 < 0;\ \rho_2 = r_1;\ \rho_3 = r_2$	Roots of P: non-physical; unstable; stable	(26)
k_i	Partial-fraction exponents	(27)
C_0	Phase-curve constant of integration	(37)
T	Venue capacity	(76)
J_g^{nat}	Natural (unconstrained) entry flow for gender g	(54)
J_f, J_m	Constrained attendance flows	(55)
Z	Revenue objective $(J_f \cdot p_f + J_m \cdot p_m)$	(61)
W	Welfare/participation objective $(J_f + J_m)$	(68)
$\mathcal{L}$	KKT Lagrangian	(64)
λ	Shadow price of quota constraint	(65)
ν_f, ν_m	Shadow prices of natural attendance bounds	(65)
κ	Proportional quota ratio	§6.4
p_f, p_m	Entry prices	(61)
ϱ_f, ϱ_m	Population fractions $(\varrho_f + \varrho_m = 1)$	(86)
$\mathcal{P}_c = 1/2$	Voronoi percolation threshold	§5
$\mathbf{s}$	Venue state vector $(f, m, G, \text{policy}, T)$	(91)
$U_{i,g}(\mathbf{s})$	Full individual utility for agent i of gender g	(91)
$u_g(r, G)$	Deterministic incentive component for gender g	(92)
$B(r)$	Scalar best-response map	(96)
$h(r) = B(r) - r$	Fixed-point residual	§6.2
α_g, β_g	Primitive utility slopes	(92)
$c_g = \Lambda_g + \Delta_g$	Total reconsideration rate for gender g	§6.1
$\mathcal{D}$, $\mathcal{V}$	Doom set; viability invariant	(105), (106)
f_N, m_N	Rescaled CTMC processes (population size N)	(100)
CTMC	Continuous-time Markov chain	§6.3
PCTL	Probabilistic Computation Tree Logic	§6.5
$\hat{\varphi}_i$, $\hat{\mu}_i$	Regression estimates of ODE coefficients	App. A

ORCID 0009-0004-3175-8962

Part I: The Aggregate Model

2. Differential Model of Naturist Venue Demographics

To our knowledge, little effort has been put into quantifying or even characterising the implications and consequences of often anecdotally discussed gender balance, or rather imbalance, observed at naturist venues. To this end, we propose a simple system of ordinary differential equations (ODEs) to model the evolution of demographics at such venues based on fixed coefficients quantifying pools of potential participants' attitudes towards the ratio of genders present.

Given f females and m males, over time evolution is posited to obey

$$\begin{cases} \frac{\mathrm{d}f(t)}{\mathrm{d}t} - \varphi_1 \cdot \frac{f(t)}{m(t)} + \varphi_2 \cdot \frac{m(t)}{f(t)} = 0 \\ \frac{\mathrm{d}m(t)}{\mathrm{d}t} - \mu_1 \cdot \frac{f(t)}{m(t)} + \mu_2 \cdot \frac{m(t)}{f(t)} = 0 \end{cases} \tag{1}$$

where φ_1 is females' preference for a higher female-to-male ratio f/m, φ_2 is females' dissatisfaction with a higher male-to-female ratio m/f, μ_1 is males' preference for a higher female-to-male ratio f/m, and μ_2 is males' dissatisfaction with a higher male-to-female ratio m/f. It is assumed, at least initially, that $\varphi_i, \mu_i \in \mathbb{R}_{>0}$[1].

In some sense the system can be interpreted as an interplay between:

(a) **"Comfort terms"** $f(t)/m(t)$, where a higher ratio of females to males generally increases the 'comfort' and perceived "social safety" for everyone attending. For females, it reduces the fear of becoming an object of unwanted attention; for males, it validates the venue as a popular, balanced social environment; and

(b) **"Pressure terms"** $m(t)/f(t)$, where a higher ratio of males to females creates a 'skewed' environment. In naturist venues, a high male-to-female ratio is often perceived as 'creepy' or 'unbalanced', leading to a decrease in the growth rate of both populations (though sensitivity can vary markedly, as discussed later).

It is assumed that *a priori* no inherent gender bias in favour or against practicing naturism exists but that observed rates of participation in public venues are driven by the gender ratios themselves. We will find that females' displeasure at being in considerable minority and males' relative indifference to high proportions of their own gender can often dominate the behaviour of the system. To state the obvious, as female proportion increases, both populations experience growth (or more precisely, an influx of participants), while as male proportion increases, both populations experience decline (or efflux).

Though such a model is the bare minimum, it suffices to capture the essence of the phenomenon while remaining broadly tractable analytically and thus serving as a source of insight, rather than quantitative accuracy that would probably be better attained by other means. Nonetheless we shall find that such a simplistic model (further extended by including finite capacity and rational decision-making by agents maximising their own utilities) has considerable descriptive power and captures many core features observed anecdotally in the field.

In the interests of brevity and clarity the dependence of f and m on t will be henceforth dropped from discussions of (1), leading to a restatement in condensed form:

$$\begin{cases} \dot{f} = \varphi_1 \cdot \frac{f}{m} - \varphi_2 \cdot \frac{m}{f} \\ \dot{m} = \mu_1 \cdot \frac{f}{m} - \mu_2 \cdot \frac{m}{f} \end{cases} \quad \text{with } \varphi_i, \mu_i \in \mathbb{R}_{>0}. \tag{2}$$

[1] These constants have units of $\left[\text{people} \cdot \text{s}^{-1}\right] = [\text{people} \cdot \text{becquerel}]$ and we shall never speak of this again.

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ORCID 0009-0004-3175-8962
DOI 10.5281/zenodo.20044325

To solve for the phase-space population trajectories, we first introduce the ratio function $r \doteq f/m > 0$ such that $f = rm$. Since the system is homogeneous and of degree zero, the phase trajectories are determined by the relationship between f and m. Using the quotient rule, we can take the derivative of r:

$$\dot{r} = \frac{\dot{f}m - f\dot{m}}{m^2} = \frac{\dot{f}}{m} - \frac{f}{m} \cdot \frac{\dot{m}}{m}. \tag{3}$$

Substituting the expressions for $\dot{f}$ and $\dot{m}$ gives:

$$\dot{r} = \frac{1}{m} \cdot \left(\varphi_1 r - \frac{\varphi_2}{r}\right) - \frac{r}{m} \cdot \left(\mu_1 r - \frac{\mu_2}{r}\right). \tag{4}$$

Factoring out $1/m$ and distributing r gives us

$$\dot{r} = \frac{1}{m} \cdot \left(-\mu_1 r^2 + \varphi_1 r + \mu_2 - \frac{\varphi_2}{r}\right). \tag{5}$$

To find trajectories in the (m, f) plane we eliminate time t by taking the ratio of the two original ODEs and then simplify by multiplying both numerator and denominator by the term f/m, and finally using $r = f/m$:

$$\frac{\mathrm{d}f}{\mathrm{d}m} = \frac{\dot{f}}{\dot{m}} = \frac{\varphi_1 \frac{f}{m} - \varphi_2 \frac{m}{f}}{\mu_1 \frac{f}{m} - \mu_2 \frac{m}{f}} = \frac{\varphi_1 (\frac{f}{m})^2 - \varphi_2}{\mu_1 (\frac{f}{m})^2 - \mu_2} = \frac{\varphi_1 r^2 - \varphi_2}{\mu_1 r^2 - \mu_2}. \tag{6}$$

Now we isolate the variables m and r to perform integration:

$$m \frac{\mathrm{d}r}{\mathrm{d}m} = \frac{\varphi_1 r^2 - \varphi_2}{\mu_1 r^2 - \mu_2} - r = \frac{\varphi_1 r^2 - \varphi_2 - \mu_1 r^3 + \mu_2 r}{\mu_1 r^2 - \mu_2}. \tag{7}$$

Separating the variables gives us:

$$\frac{\mu_1 r^2 - \mu_2}{-\mu_1 r^3 + \varphi_1 r^2 + \mu_2 r - \varphi_2} \, \mathrm{d}r = \frac{\mathrm{d}m}{m}. \tag{8}$$

The integral on the left depends on the roots of the cubic polynomial

$$P(r) \doteq r \cdot \left(\mu_2 + \varphi_1 r - \mu_1 r^2 - \frac{\varphi_2}{r}\right) = -\mu_1 r^3 + \varphi_1 r^2 + \mu_2 r - \varphi_2; \tag{9}$$

and two cases are worth distinguishing:

(a) **Fixed Rays** when $P(r) = 0$ has a positive real root r^*, the ratio $r = f/m$ is constant along the trajectory, so $\mathrm{d}r \,/\, \mathrm{d}m = 0$ and the path is the invariant ray $f = r^* \cdot m$; depending on the sign of $P'(r^*)$ this ray is *transversely* attracting or repelling (*i.e.* stable or unstable for the ratio dynamics, while motion along the ray generally continues); and

(b) **General Curves** where $\dot{r} \neq 0$ the trajectories will curve in the plane, defining the non-equilibrium dynamic flow of the naturist venue's population levels.

The one-line take-away is that in this system of ODEs modelling gender balance at naturist venues equilibria are ratios of relative populations and not specific values. Readers who wish to explore how the system responds to different combinations of parameters can do so via the interactive demonstration available on the *Wolfram Demonstrator* platform[2].

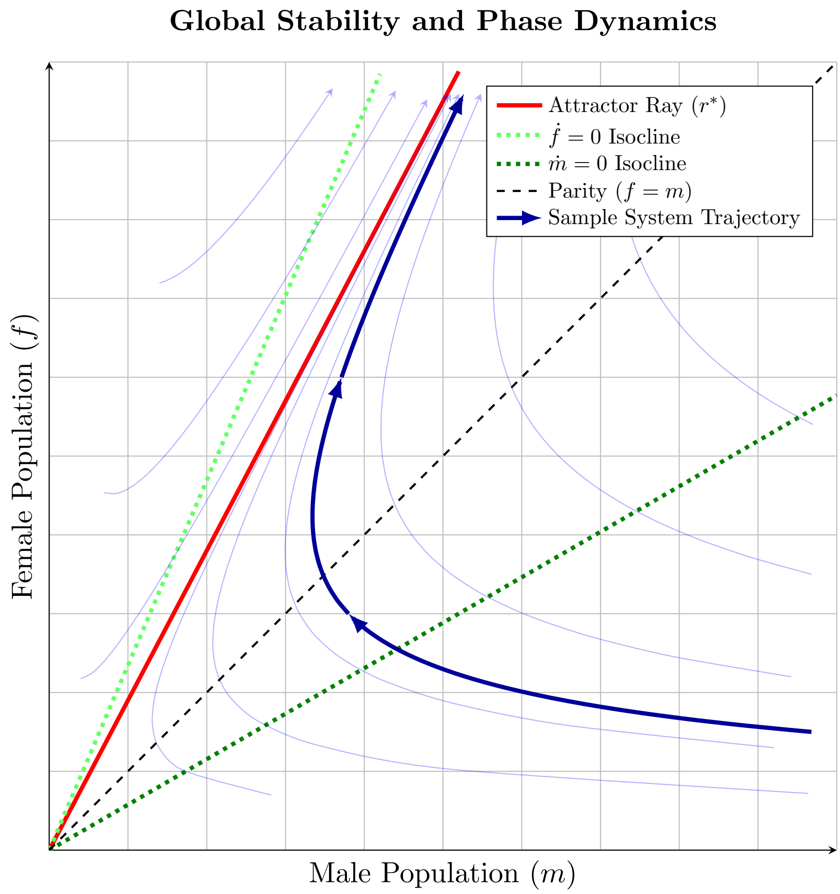

Figure 1: Global stability and phase dynamics showing streamlines converging toward the attractor ray. The attractor in this case is depicted as having a high relative proportion of females compared to males. Isoclines are rays in the phase plane: $\dot{f} = 0 \iff r = \sqrt{\varphi_2/\varphi_1}$ and $\dot{m} = 0 \iff r = \sqrt{\mu_2/\mu_1}$.

2.1. Stability of Phase-Space Solutions

Recalling (3) we have that applying the quotient rule on $r = f/m$ gives

$$\dot{r} = \frac{\dot{f}m - \dot{m}f}{m^2} = \frac{1}{m} \cdot \left(\dot{f} - r\dot{m}\right). \tag{10}$$

Substituting the system (2) yields

$$\dot{r} = \frac{1}{m} \cdot \left[\left(\varphi_1 r - \frac{\varphi_2}{r}\right) - r \cdot \left(\mu_1 r - \frac{\mu_2}{r}\right)\right]. \tag{11}$$

Combining denominators by multiplying within the bracket by r/r gives us the equation that governs the overall evolution of the gender ratio:

$$\dot{r} = \frac{1}{mr} \cdot \left(-\mu_1 r^3 + \varphi_1 r^2 + \mu_2 r - \varphi_2\right) = \frac{1}{mr} \cdot P(r). \tag{12}$$

Since the population m and the ratio r are strictly positive quantities, the sign of the rate of change of r depends entirely on the characteristic polynomial $P(r)$ from (9). The equilibrium points of the system are the roots r^* where

$$P(r^*) = 0. \tag{13}$$

 DOI 10.5281/zenodo.20044325
ORCID 0009-0004-3175-8962

The ratio dynamics (12) take the form $\dot{r} = (1/f(t)) \cdot P(r)$ on $U = (0, \infty)^2$. Because $1/f(t)$ is strictly positive it only rescales the *speed* of trajectories without reversing their direction, so the sign of $\dot{r}$ equals the sign of $P(r)$ and the local stability of a simple root r^* is determined entirely by $P'(r^*)$: the root is stable if $P'(r^*) < 0$ and unstable if $P'(r^*) > 0$.

A root $r^* > 0$ of $P(r)$ satisfies $\dot{r} = 0$ and defines an *invariant ray* $f = r^* m$ in the (m, f)-plane; we call such r^* a *ratio equilibrium.* In the unconstrained model a ratio equilibrium is generally *not* a fixed point of the full system: along $f = r^* m$ the individual populations continue to drift at rates $\dot{m} = \mu_1 r^* - \mu_2/r^*$ and $\dot{f} = \varphi_1 r^* - \varphi_2/r^*$. Generically, point-wise *state equilibrium* (f^*, m^*) where both populations are simultaneously stationary only arises once additional constraints (such as those introduced in Section 4) are imposed. The degenerate exception occurs when the compatibility condition $\varphi_2/\varphi_1 = \mu_2/\mu_1$ holds exactly, in which case $\dot{f} = \dot{m} = 0$ already in the unconstrained system; this is a measure-zero event in parameter space and not expected generically.

Not all ratio equilibria are stable. To determine the long-term fate of the venue, we analyse the stability of these roots by examining the derivative of the growth-rate $\mathrm{d}\dot{r} \,/\, \mathrm{d}r$ at the equilibrium point itself. Using the chain rule on (12):

$$\left.\frac{\mathrm{d}\dot{r}}{\mathrm{d}r}\right|_{r=r^*} = \frac{1}{mr^*} \cdot P'(r^*). \tag{14}$$

Since the prefactor $1/(mr^*)$ is always positive (both m and r^* are strictly positive), the sign of the whole expression (and therefore the stability of the equilibrium) is determined solely by the slope $P'(r^*)$:

- If $P'(r^*) < 0$ this r^* is a **stable attractor** because the polynomial slopes downwards towards this root, ensuring that any deviation will be self-correcting. This represents the venue's "social equilibrium" ratio. It is the value around which the venue's gender balance will naturally cluster.

- If $P'(r^*) > 0$ this r^* is an **unstable repulsor** because as the polynomial slopes upwards, any deviation will be self-reinforcing. This can be thought of as representing a "critical ratio". If the ratio falls beneath this critical value, the self-reinforcing feedback loop takes hold and accelerates until a single gender dominates (which we shall subsequently argue will likely be all-males).

In general, we have three distinct demographic behaviours:

- If $P(r) > 0$ then the proportion of females increases;
- If $P(r) = 0$ the system is in equilibrium and the relative proportions remain constant;
- If $P(r) < 0$ the proportion of males increases.

Let us investigate the extrema of $P(r)$ in order to characterise its behaviour and thus perform boundary analysis keeping $m, f > 0$ because the model is undefined on the axes.

As $r \to 0^+$, $\lim_{r\to 0^+} P(r) = -\varphi_2$ because all other terms are factors of r. Since φ_2 (female exit rate) is positive, this limit is negative. Demographically this implies that if the ratio is very low, $\dot{r}$ is negative, the system will be driven towards total collapse ($r = 0$) and there is no mechanism of endogenous recovery.

At $r \to \infty$ (infinite females) the leading term $-\mu_1 r^3$ dominates. Since μ_1 (male influx) is positive, $P(r) \to -\infty$. The demographic interpretation is that if somehow the female population were to become overwhelmingly predominant, a correspondingly oversized male influx will inevitably bring the ratio back down.

As the curve defined by $P(r)$ begins negative at $r = 0$ and goes to negative infinity at arbitrarily large r, there are three cases, of which two scenarios are likely for a venue:

- No positive roots imply a "**doomed venue**" where the coefficients of $P(r)$ are such that its local maximum remains below zero, and thus $\dot{r}$ is always negative. Demographically this entails that irrespective of how many females attend initially, the ratio will relentlessly decrease until the female population vanishes.

- Two positive roots imply a "**viable venue**" where the attraction parameters (φ_1, μ_2) are strong enough relative to the "exit/entry" pressures to make the cubic curve bulge upwards and cross the axis twice at roots that we shall denote r_1 and r_2. We can characterise these roots on the basis of the gradient of $P(r)$.

- The repeated (double) root corner-case implies a "**dicey venue**". This occurs when $P(r)$ is tangent to the axis at a positive ratio $r^* > 0$, *i.e.*

  $$P(r^*) = 0 \quad \text{and} \quad P'(r^*) = 0, \tag{15}$$

  so r^* is a *double root*. Equivalently, P has a saddle-node (fold) at r^* and can be factored as

  $$P(r) = -\mu_1 \, (r - r^*)^2 \, (r - \rho_1), \qquad \rho_1 < 0. \tag{16}$$

  Locally, a Taylor expansion about r^* gives

  $$P(r) = \tfrac{1}{2} P''(r^*) \, (r - r^*)^2 + O\big((r - r^*)^3\big) \, . \tag{17}$$

  Because r^* is the degenerate local maximum at the tangency point, $P''(r^*) < 0$. Defining the *curvature coefficient*

  $$A \doteq -\tfrac{1}{2} P''(r^*) > 0, \tag{18}$$

  the leading-order behaviour is therefore

  $$P(r) \approx -A \, (r - r^*)^2, \tag{19}$$

  so $P(r) \leq 0$ on both sides of r^* (strictly negative for $r \neq r^*$). Hence $\dot{r} = P(r)/(mr)$ is non-positive in a punctured neighbourhood of r^*: the ratio is not restored toward r^* from either side, and the case is so structurally unstable to even the smallest perturbation that we would not expect to observe such a scenario as it they are likely absolutely transient.

The lower root r_1 is a "viability threshold" where the curve makes its first crossing from negative to positive territory. Its slope is positive and this is an *unstable repulsor*, and the dynamics are as follows:

- If the system begins slightly below this root ($r < r_1$), $P(r)$ is negative and $\dot{r} < 0$ so the system will be pushed down towards zero (collapse of the female population).

- If the system begins slightly above this root ($r > r_1$), $P(r)$ is positive and $\dot{r} > 0$ so the system will be pushed upwards towards the second root. This is the "critical ratio" the venue must stay above in order to remain viable.

The upper root r_2 is a "natural equilibrium" where the curve would make a second crossing from positive territory back into negative values. The slope here is negative and it is a *stable attractor*: if the system is above this value, $\dot{r} < 0$ (the ratio decreases back towards equilibrium), and if the system is below this value, $\dot{r} > 0$ (the ratio increases back towards equilibrium). This represents the long-term destiny of the venue: as long as the ratio stays above the lower threshold r_1, the demographics will naturally settle here.

Summarising the stability analysis, the general condition for a sustainable naturist venue in this model is that the polynomial $P(r)$ must have a local maximum greater than zero. We can find the local maximum by solving

$$P'(r) = -3\mu_1 r^2 + 2\varphi_1 r + \mu_2 = 0. \tag{20}$$

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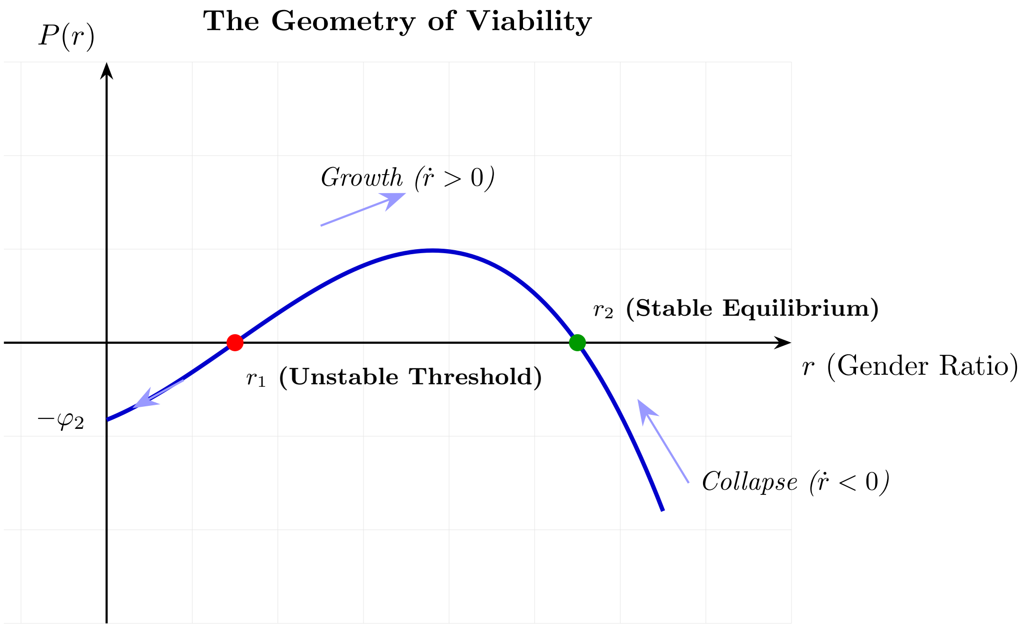

Figure 2: The Geometry of Viability: Phase Dynamics and Viability Thresholds. The cubic polynomial $P(r)$ must cross the axis to create a stable equilibrium r_2. The lower root r_1 acts as a critical threshold. Jointly they define the band within which the venue can retain a viable gender ratio.

If the value of $P(r)$ at this maximum is positive, the venue will survive and maybe thrive. If it is negative, it is destined to fail.

2.2. A compact proof of stability

Because the system is homogeneous of degree zero, stability of an equilibrium ray r^* is governed by the scalar

$$\eta(r_2^*) = \left.\frac{\mathrm{d}\dot{r}}{\mathrm{d}r}\right|_{r=r_2^*} = \frac{P'(r_2^*)}{mr_2^*} < 0. \tag{21}$$

This is geometrically equivalent to requiring that the flow of the vector field across the ray is directed inwards.

When $\varphi_2 = 0$, the cubic polynomial (9) loses its constant term and can be factored as $P(r) = r \cdot (-\mu_1 r^2 + \varphi_1 r + \mu_2)$, so immediately $P(0) = 0$; furthermore when $\varphi_2 = \mu_2 = 0$, (9) can be further factored as $P(r) = r^2 \cdot (-\mu_1 r + \varphi_1)$: intuitively enough, if efflux coefficients vanish, growth dominates as without sensitivity to pressure terms, there is nothing holding participants back.

Conversely, if the trajectory slope is steeper than any ray when above it, the gap widens. This can be fully formalised as follows: to determine if an equilibrium r^* is stable, we examine the response of the system to a small perturbation ϵ. Let $r = r^* + \epsilon$. Linearising the system around the fixed point:

$$\dot{\epsilon} \approx \epsilon \cdot \left.\frac{\mathrm{d}\dot{r}}{\mathrm{d}r}\right|_{r=r^*}. \tag{22}$$

From equation (12), noting that $P(r^*) = 0$, the derivative simplifies to:

$$\left.\frac{\mathrm{d}\dot{r}}{\mathrm{d}r}\right|_{r=r^*} = \frac{1}{mr^*} \cdot \frac{\mathrm{d}P(r)}{\mathrm{d}r}. \tag{23}$$

ORCID 0009-0004-3175-8962

Since $m, r^* > 0$, behaviour in the neighbourhood is determined strictly by the slope of the polynomial $P(r)$ at the root. Since $f(t) = m(t) \cdot r(t) > 0$ on $U = (0, \infty)^2$, the change of time variable $\mathrm{d}\tau \,/\, \mathrm{d}t = 1/f(t)$ is a strictly monotone reparametrisation; therefore the flow $\mathrm{d}r \,/\, \mathrm{d}\tau = P(r)$ has the same equilibrium types as the physical-time flow $\mathrm{d}r \,/\, \mathrm{d}t = P(r)/(mr)$, confirming that stability is determined by the sign of $P'(r^*)$ alone.

- If $P'(r^*) < 0$, then any perturbation decays, and the equilibrium is **stable**.
- If $P'(r^*) > 0$, then any perturbation grows, and the equilibrium is **unstable**.

(Readers familiar with dynamical systems will recognise this as an eigenvalue argument; the advantage here is that the scalar structure makes the stability geometry visible without matrix machinery.)

2.3. Case A: the effects of male indifference to male predominance ($\mu_2 = 0$)

Intuition guides us into believing that males have a relatively high tolerance for predominantly male demographics $\mu_2 \approx 0$. We can harness this intuition and make males totally indifferent by setting $\mu_2 = 0$. This represents a venue where males do not depart even if females are vastly outnumbered. This changes the dynamics significantly: without the "pressure relief valve" of males departing reducing the "discomfort term", the venue is much more prone to gender collapse should the female population diminish.

When $\mu_2 = 0$, the governing polynomial loses its linear term: $P(r) = -\mu_1 r^3 + \varphi_1 r^2 - \varphi_2$. Its derivative is $P'(r) = r \cdot (-3\mu_1 r + 2\varphi_1)$, so the unique positive critical point is

$$r_{\max} = \frac{2\varphi_1}{3\mu_1}, \tag{24}$$

and for this venue to remain viable this peak must be positive ($P(r_{\max}) > 0$), leading to the inequality

$$P(r_{\max}) > 0 \iff \frac{4\varphi_1^3}{27\mu_1^2} - \varphi_2 > 0 \iff \varphi_1^3 > \frac{27}{4}\mu_1^2 \varphi_2. \tag{25}$$

We are able to solve for the population trajectories in this scenario. In the viable parameter regime $P(r) = -\mu_1 r^3 + \varphi_1 r^2 - \varphi_2$ has exactly three real roots; we label them

$$\begin{cases} \rho_1 < 0 & \text{(negative, non-physical)}, \\ \rho_2 = r_1 > 0 & \text{(positive unstable)}, \\ \rho_3 = r_2 > 0 & \text{(positive stable)}. \end{cases} \tag{26}$$

The partial-fraction residue of $\mu_1 r^2 / P(r)$ at each root ρ_i is

$$k_i \doteq \frac{\mu_1 \rho_i^2}{P'(\rho_i)} = \frac{\mu_1 \rho_i^2}{-3\mu_1 \rho_i^2 + 2\varphi_1 \rho_i} \implies \frac{\mu_1 r^2}{P(r)} = \sum_{i=1}^{3} \frac{k_i}{r - \rho_i}.^2 \tag{27}$$

From (8) we then have that

$$\frac{\mathrm{d}m}{m} = \frac{\mu_1 r^2}{-\mu_1 (r - \rho_1)(r - \rho_2)(r - \rho_3)} \,\mathrm{d}r\,. \tag{28}$$

Cancelling μ_1 and noting the minus sign from the factored form of $P(r)$, the partial fraction decomposition gives:

$$\frac{\mu_1 r^2}{P(r)} = -\frac{r^2}{(r - \rho_1)(r - \rho_2)(r - \rho_3)} = \frac{k_1}{r - \rho_1} + \frac{k_2}{r - \rho_2} + \frac{k_3}{r - \rho_3}. \tag{29}$$

[2] Sign convention: k_i are defined as the residues of $\mu_1 r^2 / P(r)$, so the exponents in $m(r)$ inherit the sign of $P'(\rho_i)$ automatically; no additional minus signs are needed.

 DOI 10.5281/zenodo.20044325
ORCID 0009-0004-3175-8962

Integrating both sides gives us the exact shape of the (in this case, male) curves in the phase portrait:

$$m(r) = C_0 \cdot |r - \rho_1|^{k_1} \cdot |r - \rho_2|^{k_2} \cdot |r - \rho_3|^{k_3}. \tag{30}$$

Integration details (*From the separated ODE to the phase-space law*) Starting from the separated relation (8),

$$\frac{\mu_1 r^2 - \mu_2}{P(r)}\, \mathrm{d}r = \frac{\mathrm{d}m}{m},$$

the left-hand side is a rational function of r. Assuming P has three simple roots $\{\rho_1, \rho_2, \rho_3\}$ so that $P'(\rho_i) \neq 0$, the partial-fraction form is

$$\frac{\mu_1 r^2 - \mu_2}{P(r)} = \sum_{i=1}^{3} \frac{k_i}{r - \rho_i}, \qquad k_i = \mathrm{Res}_{r=\rho_i} \frac{\mu_1 r^2 - \mu_2}{P(r)} = \frac{\mu_1 \rho_i^2 - \mu_2}{P'(\rho_i)}.$$

Integrating term-by-term:

$$\log_e m(r) = \sum_{i=1}^{3} k_i \log_e |r - \rho_i| + \text{const.} \quad \Longrightarrow \quad m(r) = C_0 \prod_{i=1}^{3} |r - \rho_i|^{k_i},$$

which is exactly the power-law product structure of (30) (Case A, $\mu_2 = 0$) and (35) (Case B). $\triangleleft$

Integration details (*Why the residues take that form*) For simple roots, the residue of $Q(r)/P(r)$ at $r = \rho_i$ follows from Taylor-expanding P near ρ_i:

$$\frac{Q(r)}{P(r)} = \frac{Q(\rho_i) + O(r - \rho_i)}{P'(\rho_i)(r - \rho_i) + O((r - \rho_i)^2)} = \frac{Q(\rho_i)}{P'(\rho_i)} \cdot \frac{1}{r - \rho_i} + O(1),$$

so $\mathrm{Res}_{r=\rho_i} Q(r)/P(r) = Q(\rho_i)/P'(\rho_i)$. In the present application $Q(r) = \mu_1 r^2 - \mu_2$. $\triangleleft$

We are remarkably also in a position to calculate $t(r)$: how long it takes a venue to reach a given gender ratio (including the special case $t(0)$, the moment of collapse). In Case A ($\mu_2 = 0$), the male rate equation simplifies to $\dot{m} = \mathrm{d}m\,/\,\mathrm{d}t = \mu_1 r$, which yields an alternative route to the time integral:

$$t(r) = \frac{1}{\mu_1} \int_{r_0}^{r} \frac{1}{\xi} \cdot \frac{\mathrm{d}m(\xi)}{\mathrm{d}\xi}\, \mathrm{d}\xi\,. \tag{31}$$

Substituting $\mathrm{d}m\,/\,\mathrm{d}\xi = m(\xi) \cdot \mu_1 \xi^2 / P(\xi)$ (valid in Case A) recovers the general formula. In both Case A and Case B the same expression is obtained directly from $\mathrm{d}t\,/\,\mathrm{d}r = mr/P(r)$:

$$t(r) = \int_{r_0}^{r} \frac{m(\xi) \cdot \xi}{P(\xi)}\, \mathrm{d}\xi\,. \tag{32}$$

Integration details (*The time-to-ratio integral*) From $\dot{r} = P(r)/(mr)$ one reads off $\mathrm{d}t\,/\,\mathrm{d}r = mr/P(r)$, so once $m(r)$ is available,

$$t(r) - t(r_0) = \int_{r_0}^{r} \frac{m(\xi) \cdot \xi}{P(\xi)}\, \mathrm{d}\xi\,.$$

Even with $m(\xi) = C_0 \prod |\xi - \rho_i|^{k_i}$, the integrand is generally non-rational (the k_i are typically irrational), so one expects a well-defined quadrature rather than a closed form. The alternative form (31) uses $\dot{m} = \mu_1 r$ (valid in Case A) as a change of variable. $\triangleleft$

For illustrative purposes, consider the following "death spiral" scenario, wherein a venue starts with an initial ratio r_0 slightly below the threshold r_1. $P(r)$ will be negative, so $\dot{r}$ will also be negative, and so the ratio begins to slide downwards. As it approaches $r = 0$, ever-smaller values of r cause the constant term $-\varphi_2$ to dominate the polynomial, and the rate of decline of $\dot{r}$ becomes approximately proportional to $1/r$. A crash finally occurs because the integral converges to a finite value, meaning that the female population doesn't just asymptotically fade away; it actually *hits zero in finite time*. The convergence of this integral (and hence the finiteness of the collapse time) ultimately rests on the male population $m(r)$ remaining finite and strictly positive as $r \to 0$, which holds in all the generic parameter regimes studied here: the male headcount does not itself vanish before the ratio does, so the integrand is well-behaved all the way to the moment of collapse.

2.4. Case B: generalisation to male sensitivity to male predominance ($\mu_2 \neq 0$)

Having derived the dynamics under the simplifying assumption $\mu_2 = 0$, we now restore male sensitivity to gender imbalance by reinstating $\mu_2 \neq 0$ in the full system (2). This constitutes a proper generalisation of the preceding section.

For a general stable equilibrium r^*, the stability condition $P'(r^*) < 0$ expands to

$$-3\mu_1(r^*)^2 + 2\varphi_1 r^* + \mu_2 < 0, \tag{33}$$

which rearranges to the stability bound

$$\varphi_1 < \frac{3\mu_1(r^*)^2 - \mu_2}{2r^*}. \tag{34}$$

Under $\mu_2 = 0$ this reduces to $\varphi_1 < \frac{3\mu_1 r^*}{2}$, recovering the Case A condition. Each unit increase in μ_2 shifts the bound downward by $\frac{1}{2r^*}$; while the permissible range of φ_1 narrows, the basin of attraction of the stable root simultaneously widens, providing an additional restoring force when the ratio deviates from r^*.

The phase-space solutions generalise accordingly. With all three roots $\{\rho_1,\ \rho_2,\ \rho_3\}$ of the full cubic $P(r)$ (now with $\mu_2 \neq 0$, generalising the Case A notation of Section 2.3) present, the populations are

$$\begin{cases} f(r) = r \cdot m(r) = r \cdot C_0 \cdot |r-\rho_1|^{k_1} \cdot |r-\rho_2|^{k_2} \cdot |r-\rho_3|^{k_3} \\ m(r) = C_0 \cdot |r-\rho_1|^{k_1} \cdot |r-\rho_2|^{k_2} \cdot |r-\rho_3|^{k_3}, \end{cases} \tag{35}$$

where the exponents generalise (27) to

$$k_i = \frac{\mu_1\rho_i^2 - \mu_2}{P'(\rho_i)} = \frac{\mu_1\rho_i^2 - \mu_2}{-3\mu_1\rho_i^2 + 2\varphi_1\rho_i + \mu_2} \quad \Longrightarrow \quad \frac{\mu_1 r^2 - \mu_2}{P(r)} = \sum_{i=1}^{3} \frac{k_i}{r-\rho_i}, \tag{36}$$

and

$$C_0 = \frac{f_0}{r_0} \prod_{i=1}^{3} |r_0 - \rho_i|^{-k_i}. \tag{37}$$

Under $\mu_2 = 0$, (36) reduces to (27), recovering the phase-space structure of (30).

The product form (30)–(35) is stated for the generic regime in which P has three simple real roots. If P has one real root and a complex-conjugate pair, the same separation of variables yields an equivalent real expression involving a logarithm of the irreducible quadratic factor and an arctan term (from the linear numerator over that quadratic); we omit that algebra since the resulting time-domain description remains parametric in either case.

Integration details (*Case B generalisation*) The Case B phase-space solutions follow the same partial-fraction route as Case A, with the single change that the numerator in the separated ODE is now $\mu_1 r^2 - \mu_2$ instead of $\mu_1 r^2$. Provided $\mu_2 \neq \mu_1\rho_i^2$ for each root ρ_i (which holds generically), the rational function $(\mu_1 r^2 - \mu_2)/P(r)$ again has simple poles at the three roots and the same integration

 DOI 10.5281/zenodo.20044325
ORCID 0009-0004-3175-8962

step applies term-by-term, yielding the modified exponents (36). The only structural difference is that $k_i = 0$ is now possible (when $\mu_2 = \mu_1 \rho_i^2$), in which case the corresponding factor $|r - \rho_i|^{k_i}$ reduces to 1 and that root disappears from the product. $\triangleleft$

The qualitative contrast between the two regimes is summarised in the table below, with stability conditions expressed at a general equilibrium r^*.

Feature	**Case A: Male Indifference** ($\mu_2 = 0$)	**Case B: Male Sensitivity** ($\mu_2 \neq 0$)
Stability condition	$\varphi_1 < \dfrac{3\mu_1 r^*}{2}$	$\varphi_1 < \dfrac{3\mu_1 (r^*)^2 - \mu_2}{2r^*}$
Female exit tolerance	**Limited**: small φ_2 is sufficient to trigger collapse	**Extended**: $\mu_2 > 0$ admits higher φ_2 without collapse
Viability threshold	**High (Fragile)**: tipping point close to equilibrium	**Low (Robust)**: tipping point pushed towards zero
Behaviour under shock	If ratio r drops slightly, males remain while females leave, triggering a “death spiral”	If ratio r drops, some males also depart, damping the fall and enabling recovery
Interpretation	Describes a casual environment of demographic fungibility	Describes a stable community with shared norms and social bonds

This analysis revalidates the presence of the $\mu_2 \cdot \frac{m}{f}$ term in the original system (1): males also shy away from ‘unbalanced’ or ‘creepy’ venues, and when they do their departure acts as a genuine self-correcting mechanism. The $\mu_2 = 0$ case, while analytically convenient, (perhaps counterintuitively) describes a worst-case scenario of extreme male indifference; incorporating $\mu_2 > 0$ yields a strictly more robust and realistic model.

2.5. Regarding Closed-form Solutions in the Time Domain

We have seen that the trajectory of the population in the phase plane is determined by the integral relation between m and r:

$$\int \frac{\mathrm{d}m}{m} = \int \frac{\mu_1 r^2 - \mu_2}{P(r)}\,\mathrm{d}r\,. \tag{38}$$

Here $P(r) = -\mu_1 r^3 + \varphi_1 r^2 + \mu_2 r - \varphi_2$ from (9) is a full cubic polynomial. In order to solve this integral analytically, we must perform a partial fractional decomposition, which in turn requires factoring the denominator into the form $-\mu_1(r-\rho_1)(r-\rho_2)(r-\rho_3)$. Though solvable in general, the symbolic expressions for these roots involve nested cube roots of complex discriminants (Cardano’s formulæ). Substituting these large symbolic expressions into the exponents of a power-law solution renders the result opaque and algebraically rather intractable. Without first quantifying the coefficients, we cannot know if the roots are real, or complex; distinct, or repeated.

This, unfortunately, is but the first and most easily dealt-with impediment. Even in fortunate or contrived circumstances where we find coefficients that yield three clean, real roots, we are immediately met with a more formidable issue: inversion.

Let us assume, for the sake of argument, that we have found a solution $m(r)$. To find time evolution $r(t)$, we must solve the integral in (32). Since $m(r)$ contains power-law factors $|r - \rho_i|^{k_i}$ with generally non-integer exponents k_i, the integrand $m(\xi) \cdot \xi / P(\xi)$ is not a rational function, and the integral does not reduce to a simple sum of logarithms. (A pure logarithmic sum would arise from $\int \mathrm{d}r\,/P(r)$ or $\int (\mu_1 r^2 - \mu_2)/P(r)\,\mathrm{d}r$, both of which are rational integrals;

the physical-time integral (32) is more complex.) Solutions therefore take the form of a generally non-elementary integral:

$$t(r) = \int_{r_0}^{r} \frac{m(\xi) \cdot \xi}{P(\xi)}\, \mathrm{d}\xi\,. \tag{39}$$

To find explicit functions $f(t)$ and $m(t)$ we must invert this equation to find r as a function of t. This integral does not admit a general elementary antiderivative, and its inversion to $r(t)$ is non-elementary. The solution $r(t)$ notionally exists but is typically only accessible via special functions or numerical quadrature. Therefore, no general closed-form formula for $f(t)$ can be given for this model. We are forced to rely on parametric descriptions of the form $(t(r), f(r), m(r))$.

Case A does not remove the inversion obstruction Setting $\mu_2 = 0$ causes the cubic to lose its linear term:

$$P(r) = -\mu_1 r^3 + \varphi_1 r^2 - \varphi_2, \tag{40}$$

which often simplifies the qualitative root structure and the partial-fraction decompositions once roots have been numerically specified. However, the central obstruction identified above persists: even when $m(r)$ and $f(r)$ admit explicit closed forms in terms of those roots, the physical-time relation

$$t(r) = \int \frac{f(r)}{P(r)}\, \mathrm{d}r \tag{41}$$

remains a non-elementary integral for the same reason as in the general case: the power-law factors in $f(r)$ mean the integrand is not a rational function, and no general closed-form antiderivative exists. The natural time-domain description therefore remains parametric in r even in Case A.

If females were also to become indifferent to being in stark minority ($\varphi_2 = 0$), the root structure simplifies further, but observation and intuition suggest that φ_2 is frequently large and significant, probably due to unfortunately not always unfounded expectations of obnoxious and even predatory male behaviour. Solutions in all these special cases would remain parametric because the inversion obstruction persists.

2.5.1. *Temporal desingularisation: a Sundman transformation approach*

We make inroads into the time-domain analysis by separating the *geometry* of the ratio dynamics from the *clock* that converts the ratio evolution into physical time. Let $r(t) \doteq f(t)/m(t)$ exist in $U = (0, \infty)^2$, where $f(t) = m(t) \cdot r(t)$.

From (12), the prefactor $1/[m(t) \cdot r(t)] = 1/f(t)$ is purely a *time-scaling* factor. We remove it by introducing a dual time parameter τ via the Sundman transformation[3]:

$$\frac{\mathrm{d}\tau}{\mathrm{d}t} = \frac{1}{m(t) \cdot r(t)} = \frac{1}{f(t)} \qquad \Longleftrightarrow \qquad \frac{\mathrm{d}t}{\mathrm{d}\tau} = f(\tau). \tag{42}$$

Then by the chain rule,

$$\frac{\mathrm{d}r}{\mathrm{d}\tau} = \frac{\mathrm{d}r}{\mathrm{d}t} \cdot \frac{\mathrm{d}t}{\mathrm{d}\tau} = \left(\frac{P(r)}{mr}\right) \cdot (mr) = P(r), \tag{43}$$

so the ratio dynamics reduce to a one-dimensional *polynomial flow* in dual time, with no prefactor.

2.5.2. *General solution structure*

Equation (43) is separable, so $r(\tau)$ is determined implicitly by

$$\tau - \tau_0 = \int_{r_0}^{r(\tau)} \frac{\mathrm{d}u}{P(u)}, \qquad r_0 = \frac{f_0}{m_0}. \tag{44}$$

Integration details (*The Sundman dual-time substitution*) Introducing dual time τ by $\mathrm{d}\tau\,/\,\mathrm{d}t = 1/(mr) = 1/f$, the prefactor in $\dot{r} = P(r)/(mr)$ disappears:

$$\frac{\mathrm{d}r}{\mathrm{d}\tau} = P(r) \quad \Longrightarrow \quad \tau - \tau_0 = \int_{r_0}^{r} \frac{\mathrm{d}u}{P(u)} = \sum_{i=1}^{3} \frac{\log_e |r-\rho_i|}{P'(\rho_i)} + \text{const.}$$

This gives an elementary implicit description of $r(\tau)$ even when $r(t)$ is not elementarily invertible. If P has a repeated root ρ (the saddle-node case), the partial-fraction expansion includes $(u-\rho)^{-2}$ terms; integrating then produces terms of the form $(r-\rho)^{-1}$, so $r(\tau)$ can acquire rational singularities in addition to logarithms. ◁

Once $r(\tau)$ is known, the populations are recovered by quadrature. Using (42),

$$\frac{\mathrm{d}m}{\mathrm{d}\tau} = \dot{m}\frac{\mathrm{d}t}{\mathrm{d}\tau} = \left(\mu_1 \frac{f}{m} - \mu_2 \frac{m}{f}\right) \cdot f = m \cdot (\mu_1 r^2 - \mu_2),$$

giving

$$m(\tau) = m_0\, e^{\int_{\tau_0}^{\tau} (\mu_1 \cdot r(\sigma)^2 - \mu_2)\,\mathrm{d}\sigma}, \qquad f(\tau) = r(\tau) \cdot m(\tau). \tag{45}$$

Physical time is then recovered by inverting (42):

$$t(\tau) = t_0 + \int_{\tau_0}^{\tau} f(\sigma)\ \mathrm{d}\sigma = t_0 + \int_{\tau_0}^{\tau} r(\sigma) \cdot m(\sigma)\ \mathrm{d}\sigma\,. \tag{46}$$

Lebesgue/measure formulation of the physical clock Define a positive measure on Borel sets $E \subseteq [\tau_0, \infty)$ by

$$\gamma(E) \doteq \int_E f(\tau)\ \mathrm{d}\tau\,. \tag{47}$$

Then (46) becomes $t(\tau) = t_0 + \gamma((\tau_0, \tau])$. In particular,

$$t_\infty \doteq \lim_{\tau\to\infty} t(\tau) < \infty \quad \Longleftrightarrow \quad \int_{\tau_0}^{\infty} f(\tau)\ \mathrm{d}\tau < \infty, \tag{48}$$

which furnishes a clean criterion for when an "infinite dual-time" process can still terminate in finite physical time, precisely the setting of the "death spiral" discussed in Section 2.3.

2.5.3. Application to the demographic model

The male phase-space curve $m(r)$ was derived in Section 2.3 (Case A, (30)) and Section 2.4 (Case B, (35)) by separating variables in (8) and integrating term-by-term after a partial-fraction decomposition of $(\mu_1 r^2 - \mu_2)/P(r)$. The result in both cases takes the form

$$m(r) = C_0 \cdot |r-\rho_1|^{k_1} \cdot |r-\rho_2|^{k_2} \cdot |r-\rho_3|^{k_3}, \tag{49}$$

with exponents k_i given by (36) and constant C_0 by (37). Since $f = rm$ the female curve follows directly:

$$f(r) = C_0 \cdot r \cdot |r-\rho_1|^{k_1} \cdot |r-\rho_2|^{k_2} \cdot |r-\rho_3|^{k_3}. \tag{50}$$

The complete phase-space system is therefore

$$\begin{cases} f(r) = C_0 \cdot r \cdot |r-\rho_1|^{k_1} \cdot |r-\rho_2|^{k_2} \cdot |r-\rho_3|^{k_3} \\ m(r) = C_0 \cdot |r-\rho_1|^{k_1} \cdot |r-\rho_2|^{k_2} \cdot |r-\rho_3|^{k_3}, \end{cases} \tag{51}$$

where C_0 is fixed by the initial condition $r_0 = f_0/m_0$.

Integration details (*The female curve from the male curve*) Taking logarithms of $f = rm$ gives $\log_e f = \log_e r + \log_e m$, so

$$\frac{\mathrm{d}f}{f} = \frac{\mathrm{d}r}{r} + \frac{\mathrm{d}m}{m} = \left(\frac{1}{r} + \sum_{i=1}^{3} \frac{k_i}{r-\rho_i}\right) \mathrm{d}r\,,$$

where the second equality substitutes $\mathrm{d}m\,/m = \sum k_i\ \mathrm{d}r\,/(r-\rho_i)$ from (36). Integrating gives the product form (50) directly. As a useful consistency check, one can verify the same result independently by working from the female ODE: dividing $\dot{f}$ by $\dot{r}$ gives

$$\frac{\mathrm{d}f}{f} = \frac{\varphi_1 r^2 - \varphi_2}{r \cdot P(r)}\ \mathrm{d}r\,,$$

and the algebraic identity $(\varphi_1 r^2 - \varphi_2) - P(r) = r \cdot (\mu_1 r^2 - \mu_2)$ reduces this to $(1/r + \sum k_i/(r-\rho_i))\ \mathrm{d}r$, confirming that the female ODE independently yields the same exponents $k_i = (\mu_1 \rho_i^2 - \mu_2)/P'(\rho_i)$.[3] $\triangleleft$

In dual time, the ratio evolves as $\mathrm{d}\tau = \mathrm{d}r\,/P(r)$, so inverting (42) gives

$$\mathrm{d}t = \frac{f(r)}{P(r)}\,\mathrm{d}r\,, \tag{52}$$

and hence the time-domain integral

$$t(r) = \int \frac{C_0 \cdot r \cdot |r-\rho_1|^{k_1} \cdot |r-\rho_2|^{k_2} \cdot |r-\rho_3|^{k_3}}{-\mu_1 r^3 + \varphi_1 r^2 + \mu_2 r - \varphi_2}\,\mathrm{d}r\,. \tag{53}$$

This integral does not admit a general elementary antiderivative, confirming the conclusion of the preceding paragraphs.[4] It nonetheless provides a well-defined parametric description $(t(r), f(r), m(r))$ of every trajectory in the system, and is amenable to numerical quadrature once coefficients are specified.

3. Forcing Viability: the "*No Single Males*" policy

We have so far discussed "free entry" models, where the influx of females and males were independent functions of the current ratio r with natural desire to attend given by:

$$\begin{cases} J_f^{nat} \doteq \varphi_1 r, \\ J_m^{nat} \doteq \mu_1 r. \end{cases} \tag{54}$$

To formalise the "no single males" policy sometimes enforced by certain venues, we will model the venue as an optimiser facing a binding constraint. The policy dictates that female attendance must equal or exceed male attendance $J_f \geq J_m$. In the strictest and most draconian interpretation, that would involve implementing a patently absurd policy whereby a potentially random male attendee would be evicted whenever a female attendee chooses to leave the venue. Instead, we force $\dot{f}_{in} = \varphi_1 r \geq \dot{m}_{in} = \mu_1 r$, thus creating a system with a binding constraint *entry-flow* inequality, and as instantaneous headcount ratio $r = f/m$ may lag the entry rule whenever exit terms are spontaneous and physiological, so strictly the two should not be conflated, but for the sake of tractability we shall treat them as one and the same. Formally, we assume

[3] This identity $\mathrm{d}f\,/f - \mathrm{d}m\,/m = \mathrm{d}r\,/r$ is simply the logarithmic derivative of $r = f/m$.

[4] In principle, special-function reductions can occur for exceptional parameter choices. Elliptic integrals arise when the physical-time integrand can be rewritten as a rational function of r and $\sqrt{Q(r)}$ with Q cubic or quartic (equivalently, when the integrand exhibits purely square-root branching at the roots of P). In the present model this would require the phase-space exponents $k_i = (\mu_1 \rho_i^2 - \mu_2)/P'(\rho_i)$ to fall into a half-integer pattern (after any cancellations), which is non-generic. For generic coefficients the k_i are not half-integers and the physical-time integral belongs instead to the broader class of algebraic/hypergeometric integrals, so a parametric or numerical treatment remains the natural description.

a fast-mixing (short dwell-time) regime in which the stock ratio tracks the admission-cohort ratio up to a small lag over a few turnover cycles. In practice the distinction matters most during rapid transitions: a venue that enforces the entry rule at the door will see its headcount ratio converge to the same bound within a few turnover cycles, so the two are empirically close whenever people do not linger for very long periods relative to the pace at which the venue fills.

In this scenario of constraint dynamics, evolution of the male population is now governed by the minimum of "natural influx" and thus "policy limit":

$$\dot{m} = \min\left(\mu_1 r, \varphi_1 r\right) - \frac{\mu_2}{r}. \tag{55}$$

Assuming the venue is popular enough that male demand is unconstrained and thus wish to enter at a higher rate than females do (*i.e.* $\mu_1 > \varphi_1$ which is anecdotally not unreasonable and the root cause of the instability we modelled previously), the policy constraint becomes binding.

The system of ODEs that models the system shifts to:

$$\begin{cases} \dot{f} = \varphi_1 \cdot \dfrac{f}{m} - \varphi_2 \cdot \dfrac{m}{f} \\ \dot{m} = \varphi_1 \cdot \dfrac{f}{m} - \mu_2 \cdot \dfrac{m}{f} \end{cases} \quad \text{with } \varphi_i, \mu_2 \in \mathbb{R}_{>0}. \tag{56}$$

(Note that in the expression for $\dot{m}$ the coefficient μ_1 has been replaced by φ_1.)

Recall from (10) that $\dot{r} = \frac{1}{m} \cdot (\dot{f} - r\dot{m})$. Substituting the constrained rates from (56) and writing $r = f/m$ where needed:

$$\dot{r} = \frac{1}{m} \cdot \left[\left(\varphi_1 \frac{f}{m} - \varphi_2 \frac{m}{f}\right) - r \cdot \left(\varphi_1 \frac{f}{m} - \mu_2 \frac{m}{f}\right)\right]. \tag{57}$$

This simplifies to the modified version of (12), the equation that governs the overall gender ratio when the constraint is binding:

$$\dot{r} = \frac{1}{mr} \cdot \left(-\varphi_1 r^3 + \varphi_1 r^2 + \mu_2 r - \varphi_2\right). \tag{58}$$

The original destabilising cubic term $-\mu_1 r^3$ has been replaced by the 'policy' term $-\varphi_1 r^3$. As we generally assume $\mu_1 > \varphi_1$ (males are more eager to attend than females), replacing μ_1 with φ_1 significantly flattens the downward curve of the cubic polynomial $P_{policy}(r)$. This pushes roots towards the left, effectively lowering the critical threshold required for viability. ***Essentially, by tying male entry to female entry, policymakers artificially slow the rate of male growth to match the female 'comfort' growth rate and ensure the ratio is always viable by rationing male attendance.*** The venue cannot become 'oversaturated' with males by the process of entry, only via differential *physiological* exit (which, as previously mentioned, we acknowledge to be a limitation of the approach). As we have seen, μ_2 is frequently low enough to be well approximated by $\mu_2 \approx 0$, and this may be an issue.

3.1. Shadow Costs: Unlocking the Value of Others

In the unconstrained region (where the constraint is slack and not effective) the 'cost' of entry is the nominal access fee. However, when the constraint binds ($\dot{m}_{in} = \dot{f}_{in}$ despite $\mu_1 > \varphi_1$), we have *excess male demand* for admission.

There is now a difference between the number of males who wish to enter and the number who are granted access:

$$\Delta J_m = r \cdot (\mu_1 - \varphi_1). \tag{59}$$

The *shadow price* λ represents the marginal value to the venue of a female entrant, as every female who enters the venue 'unlocks' a corresponding slot for a male, and thus the presence

 DOI 10.5281/zenodo.20044325
ORCID 0009-0004-3175-8962

of a female acquires measurable economic value equal to the profit that could be generated by that unlocked male entrant.

We have already seen that unconstrained female attendance is $J_f^{nat} = \varphi_1 \cdot r$ and that unconstrained male attendance is $J_m^{nat} = \mu_1 \cdot r$ in (54), however due to policy constraints aimed at curtailing instability actual male attendance J_m cannot exceed female attendance J_f:

$$J_m \leq J_f. \tag{60}$$

In this "excess male demand" scenario, the venue is turning away males that are willing to pay in order to be granted access, depressing its revenues.

Since J_m is capped by J_f, total entry is determined solely by female demand J_f and can be at most $2J_f$. For every single female who chooses to enter, one further male is admitted. We can optimise this in two distinct manners.

3.2. Scenario A: Maximisation of Revenue (Z)

Consider the venue to be a commercial operator seeking to maximise total revenue Z (a proxy for profit and economic utility). The objective function is

$$Z \doteq J_f \cdot p_f + J_m \cdot p_m \tag{61}$$

where p_i represents access fees paid by each demographic to the venue.

To ensure the optimisation problem is well-posed and demographically meaningful, we must bound the domain. While our model still admits a venue having an infinite capacity, the market that it draws from does not. Therefore we model the venue's market as a geographic Voronoi cell defined by travel-time or travel-cost distance relative to competing venues. Within this finite catchment area with finite population, there exists a maximum number of female and male guests (f_{max} and m_{max} respectively), and thus the optimisation occurs on the compact domain $[0, f_{max}] \times [0, m_{max}]$. Consequently, our objective function is continuous on a compact set, guaranteeing the existence of a global maximum by the Extreme Value Theorem[4] ('EVT'), thereby discarding unphysical asymptotic solutions. We introduce the catchment constraints $J_f \leq J_f^{nat}$ and $J_m \leq J_m^{nat}$ and assign multipliers $\nu_f \geq 0$ and $\nu_m \geq 0$ respectively. Since a venue with zero attendance of either gender is physically vacuous and incompatible with the ratio dynamics of the model, we further note that $J_f, J_m > 0$ is required; as the global optimum identified below has $J_f^* = J_f^{nat} = \varphi_1 r > 0$ and $J_m^* = J_f^{nat} > 0$, this restriction is automatically satisfied and does not alter the domain or the EVT argument.

Identifying the global optimum Since $p_f, p_m > 0$, the objective Z is strictly increasing in both J_f and J_m. The unconstrained maximum over the box $[0, J_f^{nat}] \times [0, J_m^{nat}]$ would be the corner (J_f^{nat}, J_m^{nat}). However, since we assume $\mu_1 > \varphi_1$ (the assumption that makes the policy necessary in the first place) we have

$$J_m^{nat} = \mu_1 r > \varphi_1 r = J_f^{nat}, \tag{62}$$

so that corner violates the quota $J_m \leq J_f$. Because Z is increasing in both variables, the constrained maximum is achieved by pushing J_m as high as the quota permits ($J_m = J_f$) and then pushing J_f as high as the natural bound permits ($J_f = J_f^{nat}$). The global optimum is therefore uniquely:

$$J_m^* = J_f^* = J_f^{nat}. \tag{63}$$

At this point the quota constraint and the natural female bound are both *active*, while the natural male bound is *slack* ($J_m^* = J_f^{nat} < J_m^{nat}$).

KKT conditions at the global optimum To rigorously ground this result we invoke the Karush-Kuhn-Tucker (KKT) conditions ([5],[6]). Let $\lambda \geq 0$ be the multiplier for the quota constraint $g(J_f, J_m) = J_m - J_f \leq 0$. The Lagrangian is:

$$\mathcal{L}(J_f, J_m, \lambda, \nu_f, \nu_m) \doteq p_f J_f + p_m J_m - \lambda(J_m - J_f) - \nu_f(J_f - J_f^{nat}) - \nu_m(J_m - J_m^{nat}). \tag{64}$$

Since the objective is linear and the feasible set is a nonempty convex polytope defined by affine inequalities, KKT conditions are necessary and sufficient. Since the natural male bound is slack at the global optimum, complementary slackness gives $\nu_m = 0$. The first-order stationarity conditions then yield:

$$\begin{cases} \dfrac{\partial \mathcal{L}}{\partial J_m} = p_m - \lambda - \nu_m = p_m - \lambda = 0 \quad \Longrightarrow \quad \lambda = p_m, \\ \dfrac{\partial \mathcal{L}}{\partial J_f} = p_f + \lambda - \nu_f = 0 \quad \Longrightarrow \quad \nu_f = p_f + \lambda = p_f + p_m. \end{cases} \tag{65}$$

These conclusions are not conditional assumptions but provable consequences of the structure of the problem. The shadow prices have precise interpretations:

- $\lambda = p_m$ is the shadow price of the quota constraint: the marginal revenue of one additional admissible male entrant, and equivalently the value of relaxing the quota by one unit.
- $\nu_f = p_f + p_m$ is the shadow price of the natural female attendance constraint: the marginal value of inducing one additional female to attend naturally (relaxing J_f^{nat} by one unit). It equals her own access fee *plus* the male slot her presence unlocks.

Consequently, the effective marginal value of a female attendee to the venue is

$$V_f = \nu_f = p_f + p_m, \tag{66}$$

and the constrained maximum revenue is

$$Z^* = (p_f + p_m) \cdot J_f^{nat}. \tag{67}$$

This result provides a rigorous economic justification for cross-subsidisation schemes such as those described in [7]: any intervention that raises J_f^{nat} by one unit (*e.g.* a discounted membership tier or a social programme improving female comfort) generates marginal revenue $p_f + p_m$, and is therefore worth funding up to that value. The male premium at rate p_m above cost pays for the female subsidy at rate $\lambda = p_m$, relaxing the binding quota and increasing total revenue. To minimise lost revenue, a venue could even auction these availabilities to the highest bidder, prioritising access to those who value the experience most highly[5]. No additional assumptions are required beyond $p_f, p_m > 0$ and $\mu_1 > \varphi_1$.

3.3. Scenario B: Maximisation of Attendance (W)

Contrast this with a community-run venue seeking to maximise total participation and welfare W of its attendees, approximated here by total attendance

$$W \doteq J_f + J_m. \tag{68}$$

The optimisation problem is distinct, and the binding constraint yields the result

$$W_{constrained} \doteq 2J_f. \tag{69}$$

[5]We absolutely do not advocate in favour of this, but it is what pure economic profit-maximisation logic dictates.

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To maximise attendance and welfare, the community venue *must also* maximise female influx J_f as each additional female *still* unlocks a corresponding male vacancy.

Thus we arrive at a powerful conclusion: under the constraints of the "no single male" policy, the incentives of revenue-maximisation and welfare-maximisation are perfectly aligned. In both scenarios, the optimal strategy is invariant: the female guest is the limiting resource, and just as her preferences φ_i govern her propensity to attend, her shadow price governs the potential scale of the venue's overall attendance and acquires value as the finite resource that unlocks attendance and, through induced scarcity, valuable revenue. This equivalence is not surprising once one observes that both optimisations are subject to the same binding constraint: any mechanism that relaxes the constraint simultaneously raises both objectives. That revenue-maximisation and welfare-maximisation converge is a structural consequence of the bottleneck, not a coincidence.

3.4. Trading Off: Marginal Rates of Substitution

Marginal Rates of Substitution (MRS) describes and quantifies the trade-off required to maintain a constant level of "social utility" (or growth stability) at a venue.

Let us define a comfort proxy for female guests at the venue Θ_f as equivalent to the growth rate of female attendance (a proxy for comfort/attractiveness of the venue):

$$\Theta_f(f, m) \doteq \dot{f} = \varphi_1 \frac{f}{m} - \varphi_2 \frac{m}{f}. \tag{70}$$

The MRS is thus the answer to the question: "if we add one male (increasing discomfort), how many females must we add to keep the overall comfort level unchanged?" The iso-comfort condition $\mathrm{d}\Theta_f = 0$ requires:

$$\mathrm{d}\Theta_f = \frac{\partial \Theta_f}{\partial f}\,\mathrm{d}f + \frac{\partial \Theta_f}{\partial m}\,\mathrm{d}m = 0. \tag{71}$$

Taking the partial derivatives results in:

$$\begin{cases} \dfrac{\partial \Theta_f}{\partial f} = \dfrac{\varphi_1}{m} + \dfrac{\varphi_2 m}{f^2}, \\ \dfrac{\partial \Theta_f}{\partial m} = -\dfrac{\varphi_1 f}{m^2} - \dfrac{\varphi_2}{f}. \end{cases} \tag{72}$$

Solving for the gradient $\mathrm{d}f \,/\, \mathrm{d}m$ via the implicit function theorem gives the *substitution rate*:

$$SR_{f,m} \doteq \frac{\mathrm{d}f}{\mathrm{d}m} = -\frac{\partial \Theta_f / \partial m}{\partial \Theta_f / \partial f} = \frac{\frac{\varphi_1 f}{m^2} + \frac{\varphi_2}{f}}{\frac{\varphi_1}{m} + \frac{\varphi_2 m}{f^2}}. \tag{73}$$

Multiplying both numerator and denominator by m gives:

$$SR_{f,m} = \frac{\varphi_1 r + \varphi_2 / r}{\varphi_1 + \varphi_2 / r^2} = r \cdot \left(\frac{\varphi_1 r^2 + \varphi_2}{\varphi_1 r^2 + \varphi_2} \right) = r \tag{74}$$

and thus,

$$SR_{f,m} = \frac{f}{m}. \tag{75}$$

This should come as a surprise to nobody. In demographic terms, this means that in order to maintain the exact same level of comfort, the substitution rate is simply the current ratio. This both confirms and follows from isoclines (lines of constant growth) being rays from the origin. To keep the comfort constant, the venue must expand population linearly along the ray.

4. Full House: Finite Carrying Capacity

So far, our model has assumed unconstrained supply at infinite-capacity venues. However, as every physical venue has a hard finite total carrying capacity T (determined, amongst other things, by locker counts, sun-beds, limited accommodation or camping pitches, pool safety limits, or fire codes)[6]. Unlike biological and demographical systems where growth slows due to resource scarcity (the dampening effect of the Verhulst logistical model[8], for example), a venue usually hits a hard constraint when whatever limiting resource is fully subscribed. Furthermore, as potential attendees are neither being born nor dying according to the abundance or scarcity of resources, but are simply turned away when T is reached, it seems preferable to implement a "turnstile model".

There is an additional issue of information asymmetry that further fortifies this reasoning: naturist venues, by their very nature, tend to be private and enclosed and thus not easily viewable from outside. This entails that prospective entrants presenting themselves at the gate or attempting to make a reservation will be unaware of the current or anticipated level of attendance or crowding inside the venue's borders. Consequently only gatekeepers who are aware of the venue's occupancy rate are in a position to moderate attendance accordingly[7]. We therefore do not model a damping term on the parameters, but a boundary condition on the domain. The system evolves according to its governing equations until the total population $f + m = T$. We can formalise this as a constraint on the total number of attendees:

$$f + m \leq T. \tag{76}$$

4.1. Dynamics at Capacity: the "Churn Mechanism"

Once the venue is full and the constraint

$$f + m = T \tag{77}$$

holds, dynamics switch from growth to replacement. To make this precise, let the unconstrained vector field be

$$\begin{cases} F(f,m) \doteq \varphi_1 \dfrac{f}{m} - \varphi_2 \dfrac{m}{f}, \\ M(f,m) \doteq \mu_1 \dfrac{f}{m} - \mu_2 \dfrac{m}{f}. \end{cases} \tag{78}$$

Here F and M are the scalar-valued component functions of the drift; together they define a vector field on the population plane via $\mathbf{n}(t) \doteq (f(t), m(t))^\top$ and $\dot{\mathbf{n}} = (F(f,m), M(f,m))^\top$.

On the boundary $f + m = T$ we impose $\dot{f} + \dot{m} = 0$ (replacement rather than net growth). A turnstile rule that preserves the free-entry ratio dynamics is *proportional throttling*:

$$\begin{cases} \dot{f} = F(f,m) - \theta f, \\ \dot{m} = M(f,m) - \theta m, \\ \theta = \dfrac{F(f,m) + M(f,m)}{T}. \end{cases} \tag{79}$$

Then $\dot{f} + \dot{m} = F + M - \theta(f+m) = F + M - \theta T = 0$ holds identically on the boundary. Moreover the common throttling term cancels from the ratio equation:

$$\dot{r} = \frac{\dot{f} - r\dot{m}}{m} = \frac{(F - \theta f) - r(M - \theta m)}{m} = \frac{F - rM}{m} = \frac{1}{mr} \cdot P(r), \tag{80}$$

[6] With the notable exception (maybe) of nude beaches, but some form of physical constraint will eventually obtain.

[7] This difficulty of observation does not necessarily contradict the whole premise of the model, in that globally the model works correctly even if pressure terms act internally upon having observed the conditions and comfort terms act by word of mouth or reputation. Again nude beaches might be argued to be an exception, particularly if one is approaching from the sea.

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DOI 10.5281/zenodo.20044325

so the ratio dynamics at capacity are governed by the same polynomial $P(r)$ as in the unconstrained interior. Consequently the stable ratio equilibrium r^* with $P(r^*) = 0$ and $P'(r^*) < 0$ remains the attracting ray even in the full-house regime.

To avoid a "death spiral", demographics must settle on the stable ratio attractor ray

$$f = r^* m. \tag{81}$$

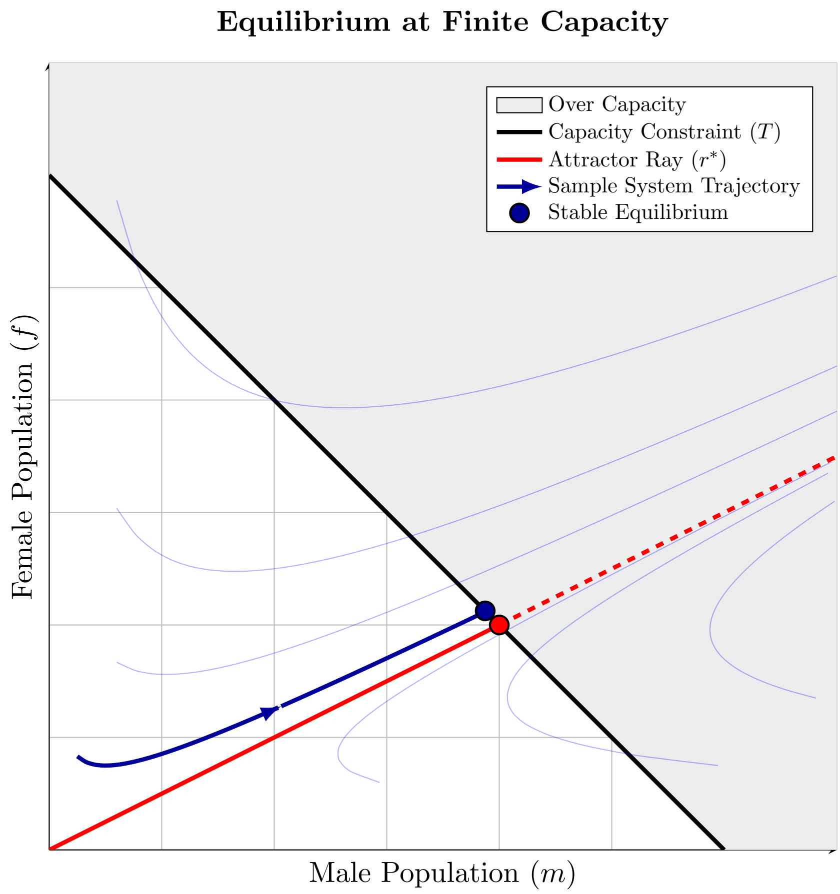

Figure 3: When capacity is constrained by a maximum T, the system intersects $f + m = T$ at the attractor ray r^*, defining the target equilibrium (f^*, m^*). Under proportional throttling the ratio continues to adjust toward r^* even at capacity; under a hard replacement rule a ratio reached at capacity may lock in.

We are going to solve for the point that satisfies both conditions simultaneously. Substituting the stable ratio constraint (81) into the capacity constraint (77) results in:

$$m + r^* m = T. \tag{82}$$

We can factor out m to obtain

$$m \cdot (1 + r^*) = T, \tag{83}$$

and solve for the male equilibrium population m^*

$$m^* = \frac{T}{1 + r^*}, \tag{84}$$

and then substitute back into (81) to find the female equilibrium:

$$f^* = \frac{T r^*}{1 + r^*}. \tag{85}$$

This point (f^*, m^*) represents the target "full occupancy" equilibrium. What happens when the venue reaches capacity at a ratio different from r^* depends critically on the replacement rule in force.

Under proportional throttling (79) The ratio dynamics retain the same sign structure $\dot{r} = \frac{1}{mr} \cdot P(r)$ on $f + m = T$, so the boundary flow drives r back toward r^* provided r lies in the basin of attraction above the unstable threshold r_1:

- If $r > r^*$ the venue is female-heavy relative to equilibrium. The ratio decreases back toward r^* as the throttle preferentially admits more males.
- If $r < r^*$ and r is already in the unstable zone below r_1, the boundary flow $\dot{r} < 0$ accelerates the decline. The venue should therefore aim to reach full occupancy at or above r^*.

Under a hard replacement rule (male-heavy, $\mu_2 \approx 0$) The proportional-throttling mechanism assumed above is an additional modelling choice. Under a stricter hard-constraint interpretation where vacancies are filled one-for-one and male exits are negligible ($\mu_2 \approx 0$), the venue may face a ratio lock-in. Since at capacity every female vacancy must be replaced by another female who cannot be accompanied (as that would make r more male-dominated still and furthermore exceed capacity T by one unit), $\dot{f}_{\text{in}} = \dot{f}_{\text{out}}$ and $\dot{m}_{\text{in}} = 0$, giving $\dot{r} = 0$: replacing an unhappy female with another unhappy unaccompanied female does not change r. Under this rule, *capacity can freeze a bad ratio.* If the venue hits T at a male-heavy $r < r^*$, the ratio locks in and may not self-correct. Management must therefore either prevent hitting T at male-heavy r, or induce non-negligible male outflow, or operate intentionally below capacity.

Part II: Microfoundations and Rational Agency

5. The Topology of Comfort

To understand why the specific constraint $r \geq 1$ is so frequently advocated, we must look beyond aggregate demographics and examine the detailed social topology of the venue. The 'comfort' of a female guest, and specifically her ability to remain largely indifferent to the males present ($\varphi_2 \approx 0$) is likely not merely a psychological trait or social convention[8], but an indicator of her local connectivity to other attendees and her social horizon.

We can model the venue floor as a stochastic Voronoi tessellation[9]. A "safe space" for a female guest can be tentatively defined topologically: she feels secure if she is part of a couple or if she belongs to a connected component of other female guests large enough to form a "social shield" against isolation or encirclement. This maps the problem to percolation theory.

Maintaining Sightlines

Case when $r > 1$: Connected female network | **Case when $r < 1$: Shattered female network**

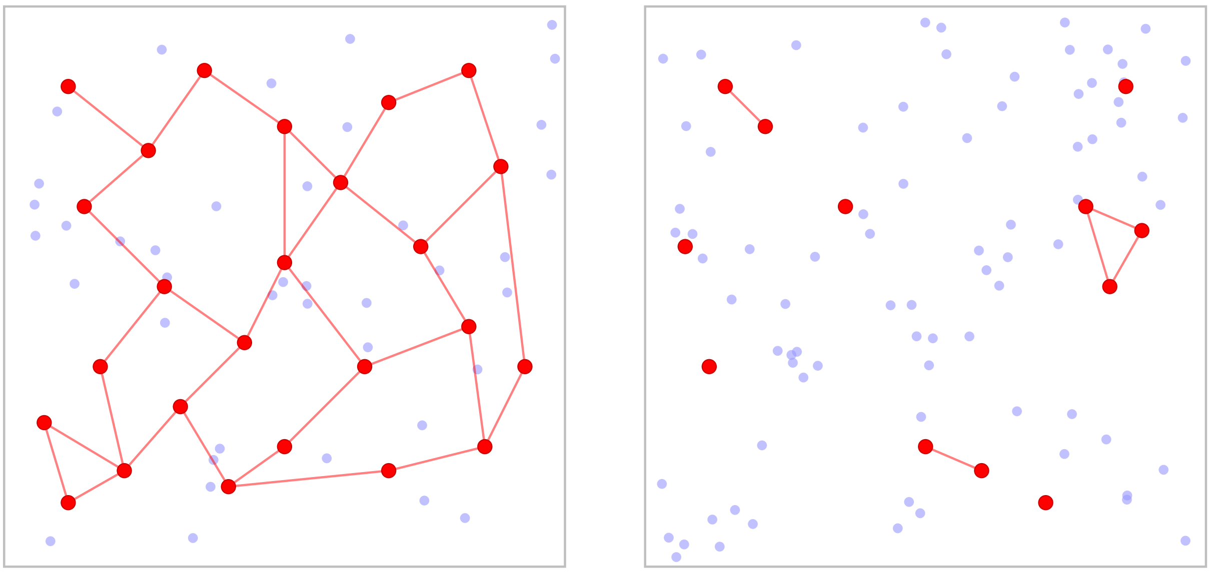

Figure 4: By analogy with the Voronoi percolation threshold, a ratio $r > 1$ ensures $\varrho_f = r/(1+r) > 1/2$, placing the venue above the heuristic connectivity threshold and enabling a connected female social graph providing "comfort in numbers". Below $r = 1$ that network may shatter into isolated clusters.

It is a known result [10] in stochastic geometry (see also [11] for the broader context of dynamical processes on networks) that the critical probability threshold $\mathcal{P}_c$ for the emergence of a "giant component" (a connected subgraph) on a Voronoi tessellation is $\mathcal{P}_c = 1/2$. That theorem applies to an infinite homogeneous plane, whereas a naturist venue is finite, spatially

[8]Though one might argue that a female who considers attending a naturist venue is, by that very fact, already comfortable with circumstances broader than mainstream social convention.

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DOI 10.5281/zenodo.20044325
ORCID 0009-0004-3175-8962

heterogeneous, and socially structured, so the following should be read as an analogy rather than a derived consequence of the ODE model (though without finite capacity T one could argue that we could approximate it to an almost arbitrary degree). Nonetheless, if ϱ_f represents the probability that a random guest is female, a robust, connected female network only exists if $\varrho_f \geq 1/2$. Expressing this in terms of our ratio $r = f/m$ we find:

$$\varrho_f = \frac{f}{f+m} = \frac{r}{1+r} \geq \frac{1}{2} \iff r \geq 1. \tag{86}$$

By this analogy, $r \geq 1$ serves as a *connectivity benchmark* that motivates a modelling threshold: it is the value at which the female fraction crosses the idealised percolation critical point, providing a principled calibration for the entry-cohort constraint rather than an arbitrary rule-of-thumb. Real venues can sustain viable ratios below $r = 1$ because finite-size effects, established couples, repeated interactions, and spatial clustering all increase effective female connectivity beyond what the raw global fraction predicts. We suggest the mechanism may be as basic as females preserving lines of sight to fellow females. Since $\varrho_m = 1 - \varrho_f$, the analogy further suggests that if $\varrho_f > \varrho_m$ (*i.e.* $r > 1$), females may remain confident that males are unlikely to form a "giant male component" of their own. At $r = 1$ exactly, $\varrho_f = \varrho_m = 1/2$, meaning the two networks are symmetrically balanced in the idealised model. Thus $r \geq 1$ can be understood as a heuristic benchmark for the topological integrity of the female social graph. Below this ratio the female network may undergo a transition into isolated, vulnerable clusters in this analogy, likely elevating the effective φ_2 as a defensive mechanism[9].

5.1. When women are at ease

We have stated that it is fairly intuitive to assume that $\varphi_2 \gg 0$ because, anecdotally, females may be made uneasy by expectations of inappropriate male behaviour and therefore seek "comfort in numbers". When attending as a couple, $\varphi_2 \approx \mu_2 \approx 0$, presumably because companionship brings its own sense of security.[10] For economy of argument, let us now return to the free-entry model, as the effects of finite supply have largely been understood. Recall that gender ratio dynamics are governed by the cubic (9) with equilibria at (13). Differentiating the equilibrium condition $P(r_i, \varphi_2) = 0$ implicitly with respect to φ_2 (valid by the implicit function theorem wherever $P'(r_i) \neq 0$; the boundary case $P'(r_i) = 0$ is precisely the "dicey venue" saddle-node) gives, using $\partial P/\partial\varphi_2 = -1$:

$$\frac{\mathrm{d}r_i}{\mathrm{d}\varphi_2} = -\frac{\partial P/\partial\varphi_2}{P'(r_i)} = \frac{1}{P'(r_i)}. \tag{87}$$

This has a remarkable consequence: a rising φ_2 causes the two positive roots to move towards each-other until they coalesce into a saddle-node at the boundary case where $P(r^*) = 0$ and $P'(r^*) = 0$ simultaneously (equivalently, the discriminant of the cubic vanishes). When $\varphi_2 = 0$ we have that for $P(0) = 0$ and a small $r > 0$ the following approximation holds:

$$P(r) \approx \mu_2 r + \varphi_1 r^2 > 0, \tag{88}$$

which in turn means that in these circumstances $\dot{r} > 0$ and thus ***the gender ratio r is pushed upwards rather than being sucked into collapse***, and indeed the reader can verify that

$$\varphi_2 = 0 \iff P(r) = -\mu_1 r^3 + \varphi_1 r^2 + \mu_2 r = r \cdot (-\mu_1 r^2 + \varphi_1 r + \mu_2). \tag{89}$$

[9] Which we shall very briefly touch upon later.

[10] In the interest of intellectual honesty we note that an alternative micro-mechanism could yield the same reduced-form coefficient: male indifference to a companion's discomfort, whereby a male remains at the venue (keeping μ_2 low) even when his female partner experiences high pressure, sustaining a low effective φ_2 through the couple unit rather than through female-specific shielding. We seriously doubt this mechanism describes the typical case: couples in our sense attend together precisely because their preferences are aligned, and a male who would remain while his partner is in distress is unlikely to attend as part of a stable couple unit in the first place. We mention the alternative only to acknowledge that the reduced-form coefficient does not, by itself, distinguish between the two micro-stories.

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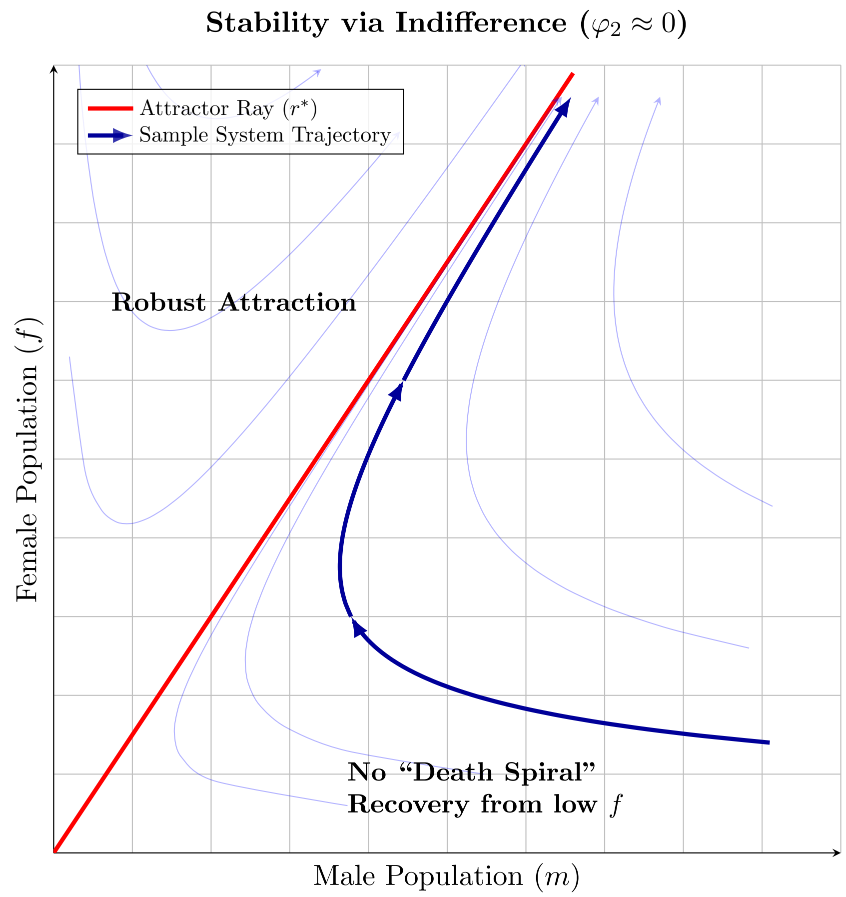

Figure 5: When $\varphi_2 \approx 0$ the constant term of $P(r)$ vanishes, placing a root at $r = 0$ and reversing the sign of $\dot{r}$ for small positive r. The phase flow no longer pulls the ratio toward collapse at the origin; the venue exhibits robust attraction even from low female counts.

Thus we have that for small $r > 0$, $P(r) \approx \mu_2 r > 0$ and $\dot{r} > 0$: a welcome impediment to the "death spiral". Mathematically, when $\varphi_2 = 0$ the constant term of $P(r)$ vanishes, placing a root at $r = 0$ and reversing the sign of P for small positive r; the system is freed from the negative basin that otherwise drags the ratio toward collapse. Note that this does not alter the homogeneity of degree zero of the underlying ODE system, nor does it move the attractor rays; what changes is the local dynamics of $P(r)$ near zero, which no longer pull the ratio downward when $\varphi_2 = 0$. Demographically this frees the venue from the "tyranny of the origin": a small perturbation below any positive ratio is no longer inevitably fatal.

This explains why small, high-trust groups (such as a few heterosexual couples) at a venue can easily accommodate a ratio-breaking event such as the arrival of a single male. φ_2 is a coefficient of female sensitivity to minority status, but first and foremost a measure of justified concern with inappropriate male behaviour, which is why couples exhibit $\varphi_2 \approx \mu_2 \approx 0$: the female in a couple feels safe from inappropriate attention. If a norm of male propriety could be strictly enforced, the system can literally "get off to a good start" where high trust and ease create an environment far more robust to imbalance than the base model suggests[11]. Combined, the two effects explain why amongst strangers females prefer a ratio that does not disfavour them heavily, while in a group they can form a stable comfort network, and in an environment of trust the system loses the characteristic sensitivity that had appeared overly prudential.

[11] This is not to be confused with the "*dicey venue*" third case described earlier, and is in many ways its opposite.

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DOI 10.5281/zenodo.20044325
ORCID 0009-0004-3175-8962

6. Microfoundations: Individual Decisions and the Mean-Field Limit

6.1. Agents Discretely make Useful Choices

The natural question to ask of the ODE system (2) is: what must an individual agent's preferences look like for the aggregate dynamics to take exactly that form? It turns out that the answer is almost uniquely determined, and computable.

To connect individual decisions to the aggregate model we make one simplifying assumption about the participation pool: we treat the number of prospective attendees outside the venue as large relative to attendance, so that each person's decision to join or leave depends on the current gender composition but not on how many others happen to be deliberating at the same moment. This is the naturist-venue analogue of the large-market assumption standard in discrete-choice economics, and it is what makes the net flow depend only on the ratio r rather than on absolute headcounts, giving the same degree-zero homogeneity as the ODE system (2). Venues with strong occupancy-dependent effects (a naturist beach which, particularly when approached from the seaboard, looks crowded) would require a richer specification.

Let Λ_g be the rate at which an absent agent considers joining and Δ_g the rate at which a present agent considers leaving. The total rate of participation reconsideration for an agent of gender $g \in \{f, m\}$ is then

$$c_g \doteq \Lambda_g + \Delta_g > 0. \tag{90}$$

We calibrate the outside option so that $\Lambda_g = \Delta_g = c_g/2$: in the absence of any incentive there is no net drift. The venue state is the vector $\mathbf{s} = (f, m, G, \text{policy}, T)$, where $G = (V, E)$ is the Voronoi social graph of current attendees and T is the capacity.

We separate two layers: (i) individual attendance incentives driving the mean-field dynamics; and (ii) venue pricing and quotas already analysed in Section 3. For clarity, we set aside venue pricing at this stage, treating any fixed access fee as absorbed into the attendee's outside option. What we are dealing with here is therefore the individual-decision layer, and it involves only the social incentives and concerns that drive attendance.

Agent i of gender g receives utility from attending:

$$U_{i,g}(\mathbf{s}) = u_g(r, G) + \varepsilon_{i,g}, \qquad \varepsilon_{i,g} \overset{\text{i.i.d.}}{\sim} \text{Logistic}(0, 1), \tag{91}$$

where $r = f/m$ and the deterministic component is

$$u_g(r, G) = \alpha_g r - \frac{\beta_g}{r} + \kappa_g \cdot \mathbf{1}\{\mathsf{Shield}(G, v_i)\}, \qquad \alpha_g, \beta_g > 0,\ \kappa_g \geq 0, \tag{92}$$

where α_g and β_g are *primitive utility slopes.* The shield indicator $\mathbf{1}\{\mathsf{Shield}(G, v_i)\}$, evaluated at the node $v_i \in V$ representing agent i, formalises the two distinct mechanisms by which a female attendee's effective comfort term φ_2 can drop toward zero, both identified informally in Section 5:

(a) she belongs to a connected component of fellow female attendees large enough to provide "comfort in numbers" (the giant-component case from the percolation analogy), or

(b) she is present at the venue with a couple-partner (the local social-anchoring case).

To capture both formally we enrich the venue's social graph with a single distinguished vertex set: the *anchored set* $V_C \subseteq V$, comprising vertices with a stable local social tie that is independent of the global ratio. Membership in V_C can arise through several channels, of which two are most relevant in practice: vertices whose couple-partner is presently at the venue, and long-term members who function as "social hubs" for whom the venue community itself provides an equivalent anchoring tie. We do not distinguish these channels formally because they enter the dynamics symmetrically: any such vertex is locally shielded irrespective of the global ratio. The shield property is then the least fixed point of a monotone reachability operator, expressible in the modal μ-calculus [12] as:

$$\mathsf{Shield}(G, v) = \mu X.\, \big(v \in V_C \;\vee\; (v \in V_f \wedge \langle \mathrm{adj} \rangle X)\big), \tag{93}$$

where $\langle \mathrm{adj} \rangle X$ is the existential adjacency modality. Read disjunctively: v is shielded if she is locally anchored (in V_C, either by a present partner or as a long-term hub), or she is female and adjacent to another shielded vertex. The first disjunct is the local "base case" shield independent of network connectivity; the second is the recursive percolation case, restricted to female vertices via $v \in V_f$. Note that $v \in V_C$ shields any gender, which is the formal counterpart of the empirical observation that couples exhibit $\varphi_2 \approx \mu_2 \approx 0$, while the giant-component shield applies only to females, consistent with the asymmetric topology of Section 5. No counting modalities are required. The probability that a uniformly random female attendee enjoys the unmarked giant-component shield (second disjunct only) undergoes the Bollobás–Riordan percolation transition [10] at $\varrho_f = 1/2$ on the Voronoi graph, making precise the heuristic threshold $r \geq 1$ from Section 5; finite couple density on top of this raises the effective shielded fraction by the population in V_C, which is part of why high-couple venues can remain viable at $r < 1$.

Integrating out the logistic shock (setting $\kappa_g = 0$ for clarity), the probability that a randomly drawn gender-g agent attends at ratio r is:

$$x_g(r) = \sigma\!\left(\alpha_g r - \frac{\beta_g}{r}\right), \qquad \sigma(z) = \frac{e^z}{1 + e^z}. \tag{94}$$

With reconsideration rate $c_g > 0$, the net flow of gender-g agents per unit time at ratio r is:

$$\dot{n}_g(r) = c_g \cdot \left[\sigma(u_g(r)) - \tfrac{1}{2}\right]. \tag{95}$$

This is the *hazard equation*: it governs the instantaneous rate at which agents of each gender join or leave the venue as a function of the current ratio. Its fixed points (where $\dot{n}_g = 0$) are the equilibria of the attendance process, and the ratio of female to male net flow is the central quantity that connects individual decisions to the aggregate dynamics. (Note that c_g controls the *speed* of adjustment but not the equilibrium structure, since any fixed-point condition on (95) is independent of c_g.)

6.2. The Scalar Best-Response Map and its Fixed Points

Rather than work with a set-valued best-response operator on attendance sets (which, as any reader who has tried it discovers, runs into monotonicity difficulties the moment one acknowledges that adding males can lower r and reduce females' utility) we work instead with a scalar map directly on the ratio. At equilibrium, the ratio of female to male net inflow must equal the prevailing ratio itself, since otherwise the ratio would be changing.

Definition 6.1 (Net-inflow ratio map)**.** Define $B : \mathbb{R}_{>0} \to \mathbb{R}_{>0}$ by

$$B(r) \doteq \frac{c_f\left[\sigma(u_f(r)) - \frac{1}{2}\right]}{c_m\left[\sigma(u_m(r)) - \frac{1}{2}\right]}, \tag{96}$$

on the viability band where both net flows share sign (so the ratio is well-defined and positive). A *ratio equilibrium* is now cast as a fixed point $r^* = B(r^*)$. Define the *fixed-point residual*

$$h(r) \doteq B(r) - r; \tag{97}$$

a ratio equilibrium satisfies $h(r^*) = 0$, and a fixed point is *self-correcting* if $h'(r^*) < 0$. (Heuristically, $h(r)$ is the signed excess of inflow ratio over the current ratio: $h(r) > 0$ pushes r upward and $h(r) < 0$ pushes it downward, so $h'(r^*) < 0$ is simply the standard one-dimensional stability condition.)

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Lemma 6.2 (Scalar reduction)**.** In the linear-response regime $|u_g(r)| \ll 1$ (so $\sigma(z) \approx \frac{1}{2} + \frac{z}{4}$, meaning individual incentive magnitudes are small relative to the variability of preference) and with $c_f = c_m = c$:

(1) $r^* = B(r^*)$ if and only if $P(r^*) = 0$, where $P(r)$ is the characteristic polynomial (9), with the identification:

$$\begin{cases} \varphi_1 = \frac{c}{4}\alpha_f, \\ \varphi_2 = \frac{c}{4}\beta_f, \\ \mu_1 = \frac{c}{4}\alpha_m, \\ \mu_2 = \frac{c}{4}\beta_m. \end{cases} \tag{98}$$

(2) Under the mild condition

$$D(r^*) \doteq \alpha_m r^* - \beta_m / r^* > 0 \tag{99}$$

(the linear-regime male net-inflow term is positive at r^*), the sign of $h'(r^*)$ agrees with the sign of $P'(r^*)$: specifically $h'(r^*) < 0 \iff P'(r^*) < 0$. In particular, $P'(r^*) < 0$ implies the fixed point is self-correcting.

Hence the stable equilibrium $r_2 = \rho_3$ is the unique self-correcting fixed point of B in the viable regime, and the unstable threshold $r_1 = \rho_2$ is the unique self-amplifying fixed point.

Proof. We establish both parts in turn.
Part (1). In the linear regime and with $c_f = c_m$, the constant $\frac{1}{2}$ terms cancel at interior fixed points and $B(r^*) = r^*$ reduces to:

$$\frac{\alpha_f r^* - \beta_f / r^*}{\alpha_m r^* - \beta_m / r^*} = r^*.$$

Cross-multiplying and rearranging: $\alpha_f r^* - \beta_f / r^* = \alpha_m (r^*)^2 - \beta_m$. Multiplying through by r^*:

$$-\alpha_m (r^*)^3 + \alpha_f (r^*)^2 + \beta_m r^* - \beta_f = 0.$$

Dividing by the common factor $c/4$ and substituting (98) gives $P(r^*) = 0$. Every step is reversible, so the converse holds.
Part (2). Write $N(r) \doteq \alpha_f r - \beta_f / r$ and $D(r) \doteq \alpha_m r - \beta_m / r$, so $B(r) = N(r)/D(r)$ in the linear regime. At a fixed point where $N(r^*) = r^* D(r^*)$, the quotient rule gives $B'(r^*) = [N'(r^*) - r^* D'(r^*)]/D(r^*)$. Substituting the relation $\beta_f = \alpha_f (r^*)^2 - \alpha_m (r^*)^3 + \beta_m r^*$ (obtained by rearranging the fixed-point condition) into the numerator:

$$N'(r^*) - r^* D'(r^*) = 2\alpha_f - 2\alpha_m r^*,$$

giving $B'(r^*) = 2(\alpha_f - \alpha_m r^*)/D(r^*)$. Then:

$$h'(r^*) = B'(r^*) - 1 = \frac{2(\alpha_f - \alpha_m r^*) - D(r^*)}{D(r^*)} = \frac{-3\alpha_m (r^*)^2 + 2\alpha_f r^* + \beta_m}{r^* \, D(r^*)}.$$

Under $D(r^*) > 0$ and $r^* > 0$, the sign of $h'(r^*)$ is the sign of $-3\alpha_m (r^*)^2 + 2\alpha_f r^* + \beta_m$. Since $P'(r^*) = \frac{c}{4}(-3\alpha_m (r^*)^2 + 2\alpha_f r^* + \beta_m)$, $c > 0$, hence $h'(r^*) < 0 \iff P'(r^*) < 0$. □

If one considers the discrete-time best-response iteration $r_{n+1} = B(r_n)$, a standard sufficient condition for local convergence is $|B'(r^*)| < 1$. This is a stronger requirement than self-correction ($h'(r^*) < 0$) and is not in general equivalent to the ODE stability condition $P'(r^*) < 0$; the two coincide only under the additional assumption that $B'(r^*) \geq 0$.

The identification (98) resolves an important question: the ODE coefficients φ_i, μ_i encode both the utility slopes α_g, β_g and the reconsideration rate c. Two venues with identical individual preferences but different turnover rates have different ODE coefficients but the same equilibrium ratios r^*, since $P(r^*) = 0$ scales uniformly with $c/4$. The equilibrium is a preference phenomenon; the speed of approach to it is partly a turnover phenomenon.

 DOI 10.5281/zenodo.20044325
ORCID 0009-0004-3175-8962

6.3. The Mean-Field Limit

In the linear-response regime, the mean-field dynamics of the agent continuous-time Markov chain (CTMC) converge to the aggregate ODEs. Specifically, for the rescaled process indexed by population size N, we write $r_N(t) = f_N(t)/m_N(t)$ and measure the distance to the ODE trajectory:

$$\sup_{0 \le t \le T} \left\| \begin{pmatrix} f_N(t) \\ m_N(t) \end{pmatrix} - \begin{pmatrix} f(t) \\ m(t) \end{pmatrix} \right\| \xrightarrow[N \to \infty]{\mathbb{P}} 0. \tag{100}$$

Define counts $(F_N(t), M_N(t)) \in \mathbb{Z}^2_{\ge 0}$ and densities $f_N(t) = F_N(t)/N$, $m_N(t) = M_N(t)/N$. In continuous time, let the process jump by $\pm 1/N$ in the corresponding coordinate with total (population-level) rates

$$f_N \to f_N + \tfrac{1}{N} \;\text{ at rate } N\,\frac{c_f}{2}\,\sigma(u_f(r_N)), \qquad f_N \to f_N - \tfrac{1}{N} \;\text{ at rate } N\,\frac{c_f}{2}\,[1 - \sigma(u_f(r_N))], \tag{101}$$

and analogously for m_N with (c_f, u_f) replaced by (c_m, u_m), where $r_N = f_N/m_N$. The conditional drift (expected rate of change) is then

$$\mathbb{E}\left[\frac{\mathrm{d}}{\mathrm{d}t} f_N(t) \,\middle|\, f_N, m_N\right] = c_f \cdot \left[\sigma(u_f(r_N)) - \tfrac{1}{2}\right] + o(1), \tag{102}$$

and similarly for m_N, yielding the mean-field ODE in the limit.

Theorem 6.3 (Mean-field limit)**.** Scale both catchment populations by N (density scaling). Let (f_N, m_N) denote the density-scaled continuous-time Markov chain with initial conditions converging to (f_0, m_0). Then for any $T < \infty$, in the linear-response regime, the convergence (100) holds in probability uniformly on $[0, T]$, where $(f(t), m(t))$ is the solution to the ODE system (2) with coefficients identified by (98).

Proof. We verify the three conditions of Kurtz's theorem for density-dependent Markov chains [13, 14].
(i) Drift identification. In the linear regime, the mean change in f_N per unit rescaled time is:

$$\mathbb{E}[\Delta f_N \mid f_N, m_N] = c\left[\sigma(u_f(r)) - \tfrac{1}{2}\right] \approx \tfrac{c}{4}\left(\alpha_f r - \frac{\beta_f}{r}\right) = \varphi_1 r - \frac{\varphi_2}{r},$$

using $\sigma(z) \approx \frac{1}{2} + \frac{z}{4}$ and (98). The computation for m_N is identical with (φ_1, φ_2) replaced by (μ_1, μ_2).
(ii) Lipschitz continuity. The drift vector field is C^1 with bounded derivatives on any compact subset of $(0, \infty)^2$ bounded away from the axes, hence locally Lipschitz.
(iii) Bounded jumps. Each transition changes (f_N, m_N) by at most $1/N$ in each coordinate.
Conditions (i)–(iii) verify the hypotheses of Theorem 4.1 of [13], giving the stated uniform convergence in probability. □

The system of ODEs is therefore not an artifice but a theorem about the large-population limit of individually rational participation decisions in the linear-response regime. Outside this regime the correct mean-field model is the full hazard equation (95), of which the ODEs are the first-order approximation. Modelling φ_2 as a function of r with a negative slope would be a further level of sophistication entirely justified by observation and intuition; this lies squarely within the remit of this broader view and is worthy of further investigation.

Remark 6.4. The convergence result applies while the ratio remains bounded away from zero and infinity; that is, while the venue is operating within a viable range. Near total demographic collapse $(r \to 0)$, individual incentives become extreme and the logistic approximation that underlies the ODE system breaks down: an agent facing a ratio of 1:100 is not making a marginally different decision from one facing 1:10. The finite-time collapse results (the "death spiral") are therefore derived directly from the ODE system and the hazard model rather than from the large-population limit theorem, which is simply silent in that regime.

 DOI 10.5281/zenodo.20044325
ORCID 0009-0004-3175-8962

6.4. The Optimal Quota: a Three-Way Equivalence

We can now close the question the aggregate analysis left open. The welfare and revenue results of Sections 3.2 and 3.3 were derived taking $\kappa = 1$ as given. The question of why $\kappa = 1$ was not addressed. The answer turns out to be satisfyingly overdetermined: three independently motivated conditions all converge on the same answer.

A *proportional quota mechanism* is an entry rule $J_m \leq \kappa J_f$ for $\kappa > 0$. Given $\mu_1 > \varphi_1$, the quota binds at the constrained revenue optimum, giving:

$$Z(\kappa) = J_f^{nat} \cdot (p_f + \kappa p_m), \tag{103}$$

which is *strictly increasing* in κ. Revenue alone therefore pushes κ as large as possible; the ceiling must come from elsewhere. It comes from social-graph connectivity. Among agents presenting at the gate under the quota, the female fraction of the entry cohort is:

$$\varrho_f^{entry}(\kappa) = \frac{1}{1+\kappa}, \tag{104}$$

and social-graph connectivity requires $\varrho_f^{entry}(\kappa) \geq \mathcal{P}_c = 1/2$, *i.e.* $\kappa \leq 1$. We note explicitly that $\mathcal{P}_c = 1/2$ is a theorem for the idealised infinite homogeneous plane; in applying it here we treat it as a *calibrated modelling constraint* on the entry cohort, and thus the optimal quota theorem is conditional on this choice of connectivity constraint.

Normative separation Theorem 6.5 is a positive (descriptive) result about internal coherence and optimisation, conditional on adopting a proportional quota and a connectivity constraint. It does not argue that quotas are ethically justified, legally permissible, or socially desirable. Those normative questions involve rights, harms, fairness, and institutional context that sit well outside the model. The theorem should be read as: *if* a proportional quota is imposed and *if* connectivity of the entry cohort is treated as a hard requirement, *then* $\kappa = 1$ is the unique proportional quota consistent with the model's economic and dynamical structure, subject to there being two genders identified with biological sex, which is a simplification the authors found themselves forced to make for the sake of simplicity.

Theorem 6.5 (Optimal proportional quota under a connectivity benchmark)**.** Assume $\mu_1 > \varphi_1$ and $p_f, p_m > 0$. Impose a proportional quota $J_m \leq \kappa J_f$, and assume a calibrated connectivity benchmark on the entry cohort: $\varrho_f^{entry}(\kappa) = 1/(1+\kappa) \geq 1/2$. Then the revenue-maximising feasible quota is uniquely $\kappa^* = 1$.

At $\kappa^* = 1$:

(i) The KKT multipliers satisfy $\lambda = p_m$ and $\nu_f = p_f + p_m$, as derived in Section 3.2.

(ii) In the binding-quota regime, the constrained system of ODEs replaces the male entry coefficient μ_1 by φ_1, *i.e.* $\dot{m}_{in} = \varphi_1 r$: the substitution made in (56).

Proof. Revenue under a binding proportional quota is $Z(\kappa) = J_f^{nat} \cdot (p_f + \kappa p_m)$, strictly increasing in κ. The benchmark constraint $\varrho_f^{entry}(\kappa) \geq 1/2$ gives $\kappa \leq 1$. Hence the unique feasible maximiser is the upper boundary $\kappa^* = 1$.

At $\kappa = 1$ the binding quota is $J_m \leq J_f$, and the KKT system of Section 3.2 (with $\nu_m = 0$ since the natural male bound is slack) gives:

$$\begin{cases} \dfrac{\partial \mathcal{L}}{\partial J_m} = p_m - \lambda = 0 \quad \Longrightarrow \quad \lambda = p_m, \\ \dfrac{\partial \mathcal{L}}{\partial J_f} = p_f + \lambda - \nu_f = 0 \quad \Longrightarrow \quad \nu_f = p_f + p_m. \end{cases}$$

Under the binding quota, $\dot{m}_{in} = \min(\mu_1 r, \kappa \varphi_1 r) = \kappa \varphi_1 r$; at $\kappa = 1$ this equals $\varphi_1 r$, giving the stated substitution. □

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6.5. Doom, Viability, and the Modal μ-Calculus

Part I identified a "doom set" and a "viability region" informally; it is useful to give them a precise formal foundation. The language we use is the modal μ-calculus [12], a logic for describing what is reachable or avoidable in a system of transitions; in this setting, the individual join-and-leave decisions of attendees and would-be attendees. The central distinction is between two kinds of reachability: *possible* reach (there exists some sequence of decisions that leads to collapse) and *inevitable* avoidance (every possible sequence of decisions keeps the venue viable). These are captured by the "doom set" $\mathcal{D}$ and the "viability invariant" $\mathcal{V}$ respectively. The formalism is qualitative, in that it identifies which states are structurally at risk, rather than probabilistic, which would additionally ask how likely each path is; a probabilistic version would yield the same boundary r_1 but with probability weights attached.

Model the venue as a labelled transition system $(S, \to, L)$ where transitions correspond to individual join or leave events. Let $C \doteq \{\mathbf{s} : r(\mathbf{s}) < \varepsilon\}$ be the collapse region. The *doom set*, states from which collapse is possibly reachable under some sequence of decisions, is the least fixed point:

$$\mathcal{D} = \mu X.\ \big(C\ \vee\ \langle\text{step}\rangle X\big), \tag{105}$$

where $\langle\text{step}\rangle X$ reads "some one-step transition leads into X". The *viability invariant*, states from which all trajectories permanently avoid collapse, is the greatest fixed point:

$$\mathcal{V} = \nu X.\ \big(\neg C\ \wedge\ [\text{step}]\, X\big), \tag{106}$$

where $[\text{step}]\, X$ reads "all one-step transitions lead into X". Remarkably, these two sets share the same boundary.

Theorem 6.6 (Correspondence with ODE stability)**.** In the mean-field limit:

(1) Every state with $r < r_1$ lies in $\mathcal{D}$: collapse is reachable.

(2) Every state with $r < r_1$ lies outside $\mathcal{V}$: the viability invariant fails.

(3) Every state with $r > r_1$ lies in $\mathcal{V}$: all trajectories avoid collapse.

(4) The stable equilibrium ray $f = r_2 m$ is the unique attractor within $\mathcal{V}$.

(5) The tipping point $r = r_1$ is simultaneously $\partial\mathcal{D}$ and $\partial\mathcal{V}$.

Proof. We establish each statement in turn.
(1) $r < r_1 \Rightarrow \mathbf{s} \in \mathcal{D}$**.** By Section 2.1, $P(r) < 0$ for all $r \in (0, r_1)$, so $\dot r < 0$ throughout. The collapse integral $t^* = \int_0^{r_0} m(\xi) \cdot \xi/|P(\xi)|\, \mathrm{d}\xi$ converges (near $\xi = 0$, $P(\xi) \approx -\varphi_2$ gives an integrable singularity), so $r(t^*) = 0$ in finite time. Every such state enters C.
(2) $r < r_1 \Rightarrow \mathbf{s} \notin \mathcal{V}$**.** Since every trajectory reaches C in finite time, no such state can satisfy $[\text{step}]\, X$ for any $X \subseteq \neg C$.
(3) For $r \in (r_1, r_2)$: $\dot r > 0$ so r increases toward r_2. For $r = r_2$: $\dot r = 0$. For $r > r_2$: $\dot r < 0$ so r decreases toward r_2. In all cases the trajectory converges to $r_2 > 0$ and never reaches C.
(4) r_2 is the unique root with $P'(r_2) < 0$ in (r_1, ∞).
(5) r_1 is the supremum of r values outside $\mathcal{V}$ and the infimum of those inside it. ☐

What this theorem does that the aggregate analysis of Section 3 did not is to *derive* $\kappa = 1$ rather than assume it. The earlier analysis showed what follows from the constraint; the present section shows that the constraint value is uniquely determined by revenue monotonicity under the connectivity benchmark, with the KKT shadow prices and the demographic substitution following as corollaries at $\kappa^* = 1$ rather than as independent selection principles. The reader who finds the normative consequences of the quota uncomfortable (and the authors have argued that there are good reasons to) should note that the discomfort does not arise from the value $\kappa = 1$ per se, but from the decision to impose a quota at all. Once a proportional quota is on the table, $\kappa = 1$ is the only internally coherent choice.

 DOI 10.5281/zenodo.20044325
ORCID 0009-0004-3175-8962

One final point deserves emphasis. A binding quota changes not just the rate of male admission but the *composition* of marginal entrants: those turned away when $J_m = J_f^{nat}$ is a self-selected group. This selection effect can shift the effective reduced-form coefficients (for instance by screening out individuals whose behaviour would raise perceived risk to the venue's behavioural norms and tranquillity), and therefore alter the dynamical regime. The present analysis treats coefficients as fixed within each regime; empirically, a sustained policy may move parameters as well as states.

7. Conclusion

This paper presented a dynamical system and decisional approach to modelling the demographics of naturist venues. By quantifying gender-specific preferences for, and sensitivities to, population ratios, we derived an equation that governs evolution of the venue's social balance and subsequent modifications thereto.

Our analysis yields four principal theoretical conclusions:

1. **Inherent Instability:** The proverbial "invisible hand" does not guarantee a balanced gender ratio and may actively undermine venue viability. We identified the existence of a critical unstable root below which the venue enters a "death spiral" of male-dominated stagnation. This indicates that in conditions of rigid preferences, passive management is insufficient because a venue cannot organically "grow its way out" of a bad initial ratio if our intuitions about typical coefficients are correct ($\varphi_2 \gg 0, \mu_2 \approx 0$).

2. **Shadow Price of Balance:** By modelling the "no single males" policy as a binding constraint, we identified the global revenue optimum explicitly and derived KKT conditions to show that the shadow price of female attendance equals her access fee plus the male fee her presence unlocks ($\nu_f = p_f + p_m$). This provides a rigorous economic justification for cross-subsidisation (any measure that draws more female attendees naturally generates marginal revenue $p_f + p_m$), and shows that revenue maximisation (Z) and community participation (W) effectively converge on the same strategy: prioritising female attendance and participation.

3. **The Determinism of Capacity:** We showed that the target full-occupancy equilibrium (f^*, m^*) is determined by the intersection of the stable gender ratio r^* with the capacity constraint $f + m = T$. Under proportional throttling this equilibrium is attracting even at capacity. Under a hard replacement rule, however, capacity can *freeze* a bad ratio: a venue that reaches T in a male-heavy state may be unable to self-correct, making it essential that management prevent reaching full occupancy at $r < r^*$.

4. **Microfoundations and the Optimal Quota:** Section 6 derives the ODE system from first principles as the mean-field limit of individually rational discrete-choice decisions (Theorem 6.3), with the ODE coefficients identified as scaled utility slopes rather than primitives. A scalar best-response map is introduced whose fixed points are exactly the roots of $P(r)$ and whose self-correction condition ($h'(r^*) < 0$) is exactly the ODE stability condition $P'(r^*) < 0$ (Lemma 6.2). The doom and viability sets are characterised as the least and greatest fixed points of reachability and safety operators in the modal μ-calculus (Theorem 6.6), with boundary at the tipping point r_1. Finally, Theorem 6.5 establishes that if a proportional entry quota is imposed, the value $\kappa = 1$ is the unique revenue-maximising choice subject to the Voronoi percolation connectivity benchmark; the KKT shadow-price conditions and the demographic substitution of the constrained ODE then follow as corollaries at $\kappa^* = 1$, completing the justification left open by the aggregate analysis.

This model suggests that a naturist venue is not merely a static facility but a dynamic social reactor that is sensitive to several parameters and either needs to be managed or incorporate some self-regulating mechanism. It suggests that care must be exercised when priming the naturist venue with its first participants, and that seeding the initial population with couples and/or a selection of females who are largely indifferent to being outnumbered by males ($\varphi_2 \approx 0$) can go a long way towards avoiding the scenarios of gender collapse when imbalance sets in beneath the unstable threshold r_1 that would ordinarily trigger the "death spiral" process. What appears to the authors more likely than active management is that venues derive from high-trust communities that are relatively insensitive to at least small shocks to the gender ratio and that grow serenely until undealt-with male misbehaviour actually becomes a relevant issue.

7.1. Connecting the Two Levels of Description

The central thesis of this paper is that the aggregate (ODE) and individual-decision (discrete-choice) perspectives are not merely compatible but mutually reinforcing: every structural element derived from one is independently recoverable from the other. The following table makes this identification explicit.

Concept	**Aggregate ODE level (Part I)**	**Agent / fixed-point level (Part II)**
Primitives	φ_i, μ_i postulated	α_g, β_g utility slopes
Identification	Assumed	$\varphi_1 = c\alpha_f/4$ *& cetera* (Lemma 6.2)
Validity regime	Implicit	Linear response $\lvert u_g(r)\rvert \ll 1$
Equilibria	Roots of $P(r) = 0$	Fixed points $r^* = B(r^*)$
Stable equilibrium $r_2 = \rho_3$	$P'(r_2) < 0$	$h'(r_2) < 0$, *i.e.* $B'(r_2) < 1$ (self-correcting)
Unstable threshold $r_1 = \rho_2$	$P'(r_1) > 0$	$h'(r_1) > 0$, *i.e.* $B'(r_1) > 1$ (self-amplifying)
Doom set	Finite-time crash, $r < r_1$	$\mu X.\,(C \vee \langle \text{step}\rangle X)$
Viability region	$r > r_1$ (basin of r_2)	$\nu X.\,(\neg C \wedge [\text{step}]\, X)$
Tipping point	Unstable root $r_1 = \rho_2$	$\partial\mathcal{D} = \partial\mathcal{V}$
Social graph	Heuristic Voronoi analogy	Shield = μ-calculus reachability (93)
Topology effect	Separate heuristic in §5	$\kappa_g \cdot \mathbf{1}\{\mathsf{Shield}\}$ endogenous in utility
Welfare W	$J_f + J_m$ (attendance proxy)	Aggregate utility at equilibrium
Quota value $\kappa = 1$	Assumed in §3	Derived (Theorem 6.5)
Shadow price ν_f	$\nu_f = p_f + p_m$ (KKT)	Consequence of $\kappa^* = 1$

The welfare proxy $W = J_f + J_m$ is a standard reduced-form approximation to aggregate surplus under homogeneous per-capita benefit. Entry prices p_f, p_m are treated as institutional parameters; we derive the implied shadow prices under the quota rather than solving a full optimal pricing problem.

7.2. On the "*No Single Male*" policy: motivation and consequences

Some remarks about the "no single male" policy are in order, as it is notoriously contentious. Given the model and the assumptions about the coefficients that govern it, it is shown to be an effective policy to prevent collapse of the female population from which there is no endogenous return. Whether a venue chooses to adopt it or not is largely down to their values and whether the preferences of their prospective female guests make the prospect of female attendee collapse a realistic scenario that warrants guarding against. As noted above, priming the venue with couples is one such mechanism; alternatively even a small "hard-core" kernel of female participants unconcerned by males significantly outnumbering them can provide the bedrock that in turn could allow slightly more sensitive females to participate. While the mathematical model emphasises the primacy of female preferences and behaviours, it is likely that male behaviour and attitudes are a determining factor in how at ease females feel at venues and therefore how inclined they are to stay as opposed to depart when gender ratios are unbalanced in their disfavour.

The "no single male" policy is not a fundamental requirement for comfort: it is an engineering control that compensates for insufficient trust and weak norm enforcement. In high-trust settings, females' discomfort parameter effectively vanishes as does that of their male counterparts, and the venue can tolerate male-skewed ratios. In low-trust settings, the same local-connectivity condition is more fragile, so the policy appears necessary because it preserves the "giant female component" (social shielding) by restricting male access and in turn preventing the formation of a "giant male component" that may induce unease.

The model further suggests that when surrounded by a close-knit community they trust, females can accommodate growth that disfavours them provided it does not become overwhelming, or more precisely, that low effective values of φ_2 alter the local dynamics near $r = 0$ in a way consistent with that sociological interpretation. This is presumably the process by which long-standing naturist communities historically took root and endure into the present: by beginning with trust and being extended with mutual respect.

7.2.1. *Stepping back*

In closing, the authors would like to make a final remark: if the reader is uncomfortable with the rational economic consequences that derive from the "no single male" policy as described in this paper, *in primis* the emergence of a rational incentive to promote and incentivise female attendance through asymmetric pricing that makes participating females themselves "objects of pursuit", then the reader cannot consistently advocate in favour of that policy.

Though never explicit, "no single males" introduces a form of rationing and artificial scarcity, objectifying female participants by giving their presence economic value: stability is introduced but at the great social cost of making females a precious finite resource. This should cause us to pause and reflect. Perhaps the way forward is to adopt norms of male behaviour that place female participants at ease and enforce them strictly. Making females' participation a commodity to be bought (even at an invisible shadow price) is not in the interests of the broader naturist community to whom such a consequence should be antithetical to the point of rejecting the premise, as we advocate. Rather, in choosing a venue that allows the ultimate form of relaxation and self-expression, it behooves all attendees to enjoy that ease while simultaneously taking utmost care to create circumstances that ensure all other attendees are also fully at ease and can likewise enjoy the experience.

 DOI 10.5281/zenodo.20044325
ORCID 0009-0004-3175-8962

A. Statistical Estimation of Parameters

Though the model is explicitly a toy for coarsely modelling phenomena, a key question for empirical verification is whether the theoretical coefficients (φ_1,φ_2,μ_1,μ_2) can be recovered from real-world data collected in the field, and if, once estimated, they lead to observables that coincide with the results derived in this paper.

While a single observation of a venue's gender balance (observing an instantaneous r or more likely r^* because we can expect most going concerns to be in equilibrium) presents an identification problem because an infinite set of parameter combinations can yield the same root, a time-series of attendance provides sufficient variation to estimate coefficients. Recall the original model formulated in (1) and (2).

For a discrete time-series (taken at hourly or daily intervals, for example) we approximate the instantaneous rate of a gender $\dot{g}_{f\vee m}$ as the finite difference:

$$\Delta g_{t,f\vee m} \doteq g_{t+1,f\vee m} - g_{t,f\vee m}. \tag{A.107}$$

That can be arranged into a standard linear regression model of the form

$$Y \doteq \beta_1 X_1 + \beta_2 X_2 + \epsilon. \tag{A.108}$$

A.1 Estimating female coefficients φ_1 and φ_2

Let the dependent variable be the net change in females:

$$Y_{f,t} = f_{t+1} - f_t. \tag{A.109}$$

Let the independent variables be the current ratios:

$$X_{1,t} = \frac{f_t}{m_t}, \qquad X_{2,t} = \frac{m_t}{f_t}. \tag{A.110}$$

We estimate the regression

$$Y_{f,t} = \hat{\varphi}_1 X_{1,t} - \hat{\varphi}_2 X_{2,t} + \epsilon, \tag{A.111}$$

where the regression coefficient for $X_{1,t}$ gives the estimate $\hat{\varphi}_1$ and the coefficient for $X_{2,t}$ gives the estimate $-\hat{\varphi}_2$.

A.2 Estimating male coefficients μ_1 and μ_2

Similarly, let

$$Y_{m,t} = m_{t+1} - m_t. \tag{A.112}$$

We estimate

$$Y_{m,t} = \hat{\mu}_1 X_{1,t} - \hat{\mu}_2 X_{2,t} + \epsilon. \tag{A.113}$$

A.3 Data Requirements

To obtain statistically significant estimates, the data must contain **variance in the ratio** r. If, on the other hand, a venue is perfectly managed and always sits at equilibrium $r^* = f^*/m^*$, the regressors X_1 and X_2 will be unchanging constants. The same applies if observations are made in situations where there is no influx or efflux such as on a nude cruise. In these cases regression will suffer from perfect collinearity, rendering the coefficients unidentified.

Valuable data is generated when the system is out of equilibrium. An "event study" approach analysing days when the ratio is externally perturbed (*e.g.*, a bachelor or bachelorette party arrives or departs) provides a useful exogenous shock to trace out response curves (X values) and the system's correction (Y values).

A.4 Discriminating between regimes: diagnosing the effective value of μ_2

Once parameter estimates $\hat{\varphi}_1$, $\hat{\varphi}_2$, $\hat{\mu}_1$, $\hat{\mu}_2$ are in hand, the two dominant inequalities of Section 1 provide a rapid diagnostic for the regime in which a venue is operating.

Case A diagnostic (male indifference, $\hat{\mu}_2 \approx 0$). Evaluate the survival inequality (25):

$$\hat{\varphi}_1^3 \overset{?}{>} \frac{27}{4}\hat{\mu}_1^2\hat{\varphi}_2. \tag{A.114}$$

If (A.114) holds with margin, the venue is viable under the conservative $\mu_2 = 0$ model and no further refinement is necessary. If it is violated—yet the venue is observed to be stable in practice—then $\mu_2 > 0$ is providing additional stability and Case B applies.

Case B diagnostic (male sensitivity, $\hat{\mu}_2 > 0$). At the observed equilibrium r^*, evaluate the stability bound (34):

$$\hat{\varphi}_1 \overset{?}{<} \frac{3\hat{\mu}_1(r^*)^2 - \hat{\mu}_2}{2r^*}. \tag{A.115}$$

If (A.115) is satisfied while (A.114) is violated, the venue is operating in the stable Case B regime and the male-sensitivity parameter is carrying material weight in the dynamics.

The minimum effective μ_2 required to rescue a Case A-failing venue *observed at the empirical equilibrium* $r^* = 2/3$ is found directly from the Case B max-efflux bound derived in Appendix B.2: if the estimated $\hat{\varphi}_2$ exceeds the Case A threshold $\frac{4}{27}\hat{\mu}_1$ (the bound (B.7) specialised at $r^* = 2/3$), then a positive μ_2 of at least

$$\hat{\mu}_2^{\min} = 3\left(\hat{\varphi}_2 - \frac{4}{27}\hat{\mu}_1\right) \tag{A.116}$$

is required for the venue to remain viable at that observed equilibrium (see Appendix B.2; for other equilibria r^* the analogous specialisation of (34) replaces both formulae). Comparing $\hat{\mu}_2$ against $\hat{\mu}_2^{\min}$ provides a single-number verdict: if $\hat{\mu}_2 \geq \hat{\mu}_2^{\min}$, the regression is consistent with a stable Case B venue; if $\hat{\mu}_2 < \hat{\mu}_2^{\min}$, the data suggest either a misspecified model or that the venue is closer to collapse than it appears.

Because regressors include ratio terms, errors may be heteroskedastic and correlated with state variables; in practice one should use robust standard errors and, ideally, a state-space or CTMC-likelihood approach to account for measurement error and endogeneity.

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DOI 10.5281/zenodo.20044325
ORCID 0009-0004-3175-8962

B. Anecdotal observations from authoritative observers indicate male-to-female ratios of $60\% : 40\%$ are commonly observed at viable naturist venues

Nick & Lins who run the *Naked Wanderings*[15] and *Destination Clothes Free*[16] YouTube channels are well-known authorities in the naturist community. In private communication they indicated that they have frequently observed male-to-female ratios of 60% : 40% indicating $r \approx 2/3$.

It is worth noting that $r \approx 2/3$ corresponds to $\varrho_f \approx 0.4$, *i.e.* below the symmetric Voronoi percolation threshold $\mathcal{P}_c = 1/2$ discussed in Section 4. This is not necessarily a contradiction: the 1/2 threshold is the clean benchmark in the simplest symmetric model; real venues can sustain $\varrho_f < 1/2$ because clustering, established couples, repeated interactions, spatial self-segregation, and active norm enforcement all effectively increase female connectivity beyond what the raw global fraction predicts. In model terms, these factors reduce the operative φ_2 below what an isolated unaccompanied female would experience, allowing a venue to remain viable at $r^* < 1$.

Assuming $\mu_2 = 0$ in order to reduce the degrees of freedom of the system, (9) specialises to

$$P(r) = -\mu_1 r^3 + \varphi_1 r^2 - \varphi_2, \tag{B.1}$$

which in turn implies

$$P\left(\tfrac{2}{3}\right) = 0 \iff -\mu_1\left(\frac{8}{27}\right) + \varphi_1\left(\frac{4}{9}\right) - \varphi_2 = -8\mu_1 + 12\varphi_1 - 27\varphi_2 = 0. \tag{B.2}$$

and thus

$$\varphi_1 = \frac{2}{3}\mu_1 + \frac{9}{4}\varphi_2. \tag{B.3}$$

Imposing $\mu_2 = 0$ specialises the stability derivative to:

$$P'(r) = -3\mu_1 r^2 + 2\varphi_1 r. \tag{B.4}$$

The stability condition $P'(2/3) < 0$ then gives:

$$P'\left(\frac{2}{3}\right) = -3\mu_1\left(\frac{4}{9}\right) + 2\varphi_1\left(\frac{2}{3}\right) < 0 \iff -\frac{4}{3}\mu_1 + \frac{4}{3}\varphi_1 < 0 \iff \varphi_1 < \mu_1. \tag{B.5}$$

The window of viability for the typical observed venue is thus

$$\frac{2}{3}\mu_1 < \varphi_1 < \mu_1. \tag{B.6}$$

We can perform the obvious algebraic manipulations to give us the maximum tolerable female efflux under these circumstances:

$$\frac{2}{3}\mu_1 + \frac{9}{4}\varphi_2 < \mu_1 \iff \frac{9}{4}\varphi_2 < \frac{1}{3}\mu_1 \iff \varphi_2 < \frac{4}{27}\mu_1 \approx 0.148\mu_1. \tag{B.7}$$

B.1. Phase-space solutions in terms of r for observed $3 : 2$ male-to-female ratio under the $\mu_2 = 0$ assumption

Given these constraints, recalling (30) we can promptly derive the relative populations in terms of r, using the same root ordering as (26): $\rho_1 < 0$ (non-physical), $\rho_2 > 0$ (unstable), $\rho_3 = 2/3 > 0$ (stable):

$$\begin{cases} m(r) = C_0 \cdot |r - \rho_1|^{k_1} \cdot |r - \rho_2|^{k_2} \cdot \left|r - \frac{2}{3}\right|^{k_3} \\ f(r) = r \cdot m(r). \end{cases} \tag{B.8}$$

ORCID 0009-0004-3175-8962

Since $\rho_3 = 2/3$ is a known root of $P(r) = -\mu_1 r^3 + \varphi_1 r^2 - \varphi_2$ (the $\mu_2 = 0$ specialisation of (B.1)), polynomial division yields the factorisation

$$P(r) = -\mu_1\left(r - \tfrac{2}{3}\right)\left(r^2 - Sr - \frac{3\varphi_2}{2\mu_1}\right), \tag{B.9}$$

where the *strength parameter*

$$S \doteq \frac{\varphi_1}{\mu_1} - \frac{2}{3} \tag{B.10}$$

is obtained by matching the r^2 coefficient, and the constant term $-3\varphi_2/(2\mu_1)$ follows from matching the constant coefficient. Applying the quadratic formula to (B.9) gives

$$\rho_{1,2} = \frac{S \mp \sqrt{S^2 + \dfrac{6\varphi_2}{\mu_1}}}{2}. \tag{B.11}$$

The discriminant $S^2 + 6\varphi_2/\mu_1$ is positive whenever the venue is viable; the negative root is $\rho_1 < 0$ (non-physical) and the positive root is $\rho_2 > 0$ (unstable threshold).

B.2. Evaluating and comparing both cases at the observed equilibrium $r^* = 2/3$

Substituting $r^* = 2/3$ into the general stability bound (34) of Section 1.4 yields the Case B stability condition at the observed equilibrium:

$$\varphi_1 < \frac{3\mu_1(2/3)^2 - \mu_2}{2\cdot(2/3)} = \frac{\frac{4}{3}\mu_1 - \mu_2}{\frac{4}{3}} = \mu_1 - \frac{3}{4}\mu_2. \tag{B.12}$$

Setting $\mu_2 = 0$ recovers the Case A bound $\varphi_1 < \mu_1$ from (B.5).

When $r^* = 2/3$ is an equilibrium of the full system ($\mu_2 \neq 0$), $P(2/3) = 0$ imposes the constraint:

$$-8\mu_1 + 12\varphi_1 + 18\mu_2 - 27\varphi_2 = 0 \implies \varphi_1 = \frac{2}{3}\mu_1 + \frac{9}{4}\varphi_2 - \frac{3}{2}\mu_2. \tag{B.13}$$

Comparing with the Case A relation (B.3), the μ_2 term shifts φ_1 downward by $\frac{3}{2}\mu_2$: for the same equilibrium $r^* = 2/3$ to obtain, a smaller φ_1 is required once males exhibit positive sensitivity to imbalance.

Substituting the equilibrium constraint (B.13) into the stability bound (B.12) gives the maximum tolerable female efflux for Case B:

$$\varphi_2 < \frac{4}{27}\mu_1 + \frac{1}{3}\mu_2. \tag{B.14}$$

Under $\mu_2 = 0$, (B.14) reduces to the Case A bound (B.7), $\varphi_2 < \frac{4}{27}\mu_1 \approx 0.148\,\mu_1$. Positive μ_2 thus raises the ceiling on tolerable female efflux by $\frac{1}{3}\mu_2$.

The viability window for each case follows by combining the lower bound $\varphi_1 > \frac{2}{3}\mu_1 - \frac{3}{2}\mu_2$ (from requiring $\varphi_2 > 0$ in (B.13)) with the respective stability upper bound:

$$\begin{cases} \text{Case A:} & \dfrac{2}{3}\mu_1 < \varphi_1 < \mu_1, \\[2ex] \text{Case B:} & \dfrac{2}{3}\mu_1 - \dfrac{3}{2}\mu_2 < \varphi_1 < \mu_1 - \dfrac{3}{4}\mu_2. \end{cases} \tag{B.15}$$

The width of the Case A window is $\frac{1}{3}\mu_1$; the width of the Case B window is $\frac{1}{3}\mu_1 + \frac{3}{4}\mu_2$, which increases with μ_2. Positive male sensitivity thus expands the viable parameter space.

The phase-space solutions follow directly from Section 2.4 with $\rho_3 = 2/3$ substituted as the known stable root and k_i evaluated via (36). Since $\rho_3 = 2/3$ is also a root of the full cubic

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$P(r) = -\mu_1 r^3 + \varphi_1 r^2 + \mu_2 r - \varphi_2$, the same polynomial division as in Appendix B.1 applies: matching the r^2 and constant coefficients gives

$$P(r) = -\mu_1\big(r - \tfrac{2}{3}\big)\left(r^2 - Sr - \frac{3\varphi_2}{2\mu_1}\right), \tag{B.16}$$

with $S \doteq \varphi_1/\mu_1 - 2/3$ as in (B.10). The constant term $-3\varphi_2/(2\mu_1)$ is determined solely by the constant coefficients of $P(r)$ and holds for any μ_2. The remaining two roots are therefore

$$\rho_{1,2} = \frac{S \mp \sqrt{S^2 + \dfrac{6\varphi_2}{\mu_1}}}{2}, \tag{B.17}$$

identical in form to (B.11) and valid for both Case A and Case B. Setting $\mu_2 = 0$ and applying the Case A equilibrium constraint $S = 9\varphi_2/(4\mu_1)$ verifies that $6\varphi_2/\mu_1 = 8S/3$, recovering the Case A discriminant. The phase-space system is:

$$\begin{cases} m(r) = C_0 \cdot |r - \rho_1|^{k_1} \cdot |r - \rho_2|^{k_2} \cdot \left|r - \frac{2}{3}\right|^{k_3}, \\ f(r) = r \cdot m(r), \end{cases} \tag{B.18}$$

where ρ_1 and ρ_2 are given by (B.17) and the exponents k_i by (36).

The following table collects the quantitative contrast between the two cases at $r^* = 2/3$.

Quantity	**Case A** ($\mu_2 = 0$)	**Case B** ($\mu_2 \neq 0$)
Equilibrium constraint	$\varphi_1 = \frac{2}{3}\mu_1 + \frac{9}{4}\varphi_2$	$\varphi_1 = \frac{2}{3}\mu_1 + \frac{9}{4}\varphi_2 - \frac{3}{2}\mu_2$
Stability bound	$\varphi_1 < \mu_1$	$\varphi_1 < \mu_1 - \frac{3}{4}\mu_2$
Viability window	$\frac{2}{3}\mu_1 < \varphi_1 < \mu_1$	$\frac{2}{3}\mu_1 - \frac{3}{2}\mu_2 < \varphi_1 < \mu_1 - \frac{3}{4}\mu_2$
Viability width	$\frac{1}{3}\mu_1$	$\frac{1}{3}\mu_1 + \frac{3}{4}\mu_2$
Max female efflux	$\varphi_2 < \frac{4}{27}\mu_1 \approx 0.148\,\mu_1$	$\varphi_2 < \frac{4}{27}\mu_1 + \frac{1}{3}\mu_2$

The table makes concrete what Section 1.4 established in general: every quantity that governs venue viability is relaxed by a term proportional to μ_2. The Case A entries are the hard lower bounds; empirical stability beyond those bounds is evidence of a non-negligible μ_2, diagnosable via the procedure described in Appendix A.4.

James Junghanns

DOI 10.5281/zenodo.20044325
ORCID 0009-0004-3175-8962

Data Availability

This paper presents a mathematical model; no empirical dataset was generated or analysed. The interactive Wolfram Demonstrator referenced in this article is publicly available at https://wolframcloud.com/obj/james.junghanns/Published/Gender%20Balance%20At%20Naturist%20Venues.nb (preprint DOI: 10.5281/zenodo.20044325).

Funding

This research received no specific grant from any funding agency in the public, commercial, or not-for-profit sectors.

Competing Interests

The author declares no financial or institutional competing interests. In the interest of positionality, the author discloses that he is a practising naturist and member of the community being modelled. This involvement motivated the research question and informs qualitative intuitions about parameter magnitudes; it does not affect the mathematical derivations, which follow from the stated assumptions regardless of the author's community membership.

CRediT Authorship Contribution Statement

James Junghanns: Conceptualization, Formal analysis, Investigation, Methodology, Software, Validation, Visualization, Writing – Original Draft, Writing – Review & Editing. This has very much been a pet project.

Declaration of Generative AI and AI-assisted Technologies in the Writing Process

During the preparation of this work the author used Gemini (Google) to reformat the bibliography, condense prose, and assist in drafting the abstract, and Claude (Anthropic) to assist with ensuring structural coherence between Part I and Part II, which were originally two separate papers. After using these tools, the author reviewed and edited the content as necessary and takes full responsibility for the content of the publication.

Acknowledgements

I would like to thank Rachel Leigh Morgan for taking upon herself the unenviable task of reading through various revisions of this text with a keen editorial eye.

My deepest thanks also goes to Professor Giovanni Mingari Scarpello for his tireless and exacting review of the logic and the mathematics present in this paper.

Last, but not least, I wish to thank naturism promoters *extraordinaire Nick & Lins* of [15],[16] for providing their valuable feedback and the anecdotal observations made over many years of frequentation that form the core of Appendix B.

None of this would have been possible without the help provided by any of the abovementioned and they all have my deepest gratitude.

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